

## 2001 AIME I

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Question <b>1</b> Not yet answered Points out of 5	Find the sum of all positive two-digit integers that are divisible by each of their digits. Answer:
Question 2 Not yet answered Points out of 5	A finite set $S$ of distinct real numbers has the following properties: the mean of $S \cup \{1\}$ is 13 less than the mean of $S$ , and the mean of $S \cup \{2001\}$ is 27 more than the mean of $S$ . Find the mean of $S$ . Answer:
Question <b>3</b> Not yet answered Points out of 5	Find the sum of the roots, real and non-real, of the equation $x^{2001} + (\frac{1}{2} - x)^{2001} = 0$ , given that there are no multiple roots.
Question 4 Not yet answered Points out of 5	In triangle $ABC$ , angles $A$ and $B$ measure 60 degrees and 45 degrees, respectively. The bisector of angle $A$ intersects $\overline{BC}$ at $T$ , and $AT = 24$ . The area of triangle $ABC$ can be written in the form $a + b\sqrt{c}$ , where $a$ , $b$ , and $c$ are positive integers, and $c$ is not divisible by the square of any prime. Find $a + b + c$ .
Question 5 Not yet answered Points out of 5	An equilateral triangle is inscribed in the ellipse whose equation is $x^2 + 4y^2 = 4$ . One vertex of the triangle is $(0, 1)$ , one altitude is contained in the y-axis, and the length of each side is $\sqrt{\frac{m}{n}}$ , where $m$ and $n$ are relatively prime positive integers. Find $m + n$ . Answer:

Question <b>6</b> Not yet answered Points out of 5	A fair die is rolled four times. The probability that each of the final three rolls is at least as large as the roll preceding it may be expressed in the form $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Find $m + n$ .
Question <b>7</b> Not yet answered Points out of 5	Triangle $ABC$ has $AB = 21$ , $AC = 22$ and $BC = 20$ . Points $D$ and $E$ are located on $\overline{AB}$ and $\overline{AC}$ , respectively, such that $\overline{DE}$ is parallel to $\overline{BC}$ and contains the center of the inscribed circle of triangle $ABC$ . Then $DE = m/n$ , where $m$ and $n$ are relatively prime positive integers. Find $m + n$ .
Question 8 Not yet answered Points out of 5	Call a positive integer $N$ a 7-10 double if the digits of the base-7 representation of $N$ form a base-10 number that is twice $N$ . For example, 51 is a 7-10 double because its base-7 representation is 102. What is the largest 7-10 double?
Question 9 Not yet answered Points out of 5	In triangle $ABC$ , $AB = 13$ , $BC = 15$ and $CA = 17$ . Point $D$ is on $\overline{AB}$ , $E$ is on $\overline{BC}$ , and $F$ is on $\overline{CA}$ . Let $AD = p \cdot AB$ , $BE = q \cdot BC$ , and $CF = r \cdot CA$ , where $p$ , $q$ , and $r$ are positive and satisfy $p + q + r = 2/3$ and $p^2 + q^2 + r^2 = 2/5$ . The ratio of the area of triangle $DEF$ to the area of triangle $ABC$ can be written in the form $m/n$ , where m and $n$ are relatively prime positive integers. Find $m + n$ . Answer:
Question <b>10</b> Not yet answered Points out of 5	Let $S$ be the set of points whose coordinates $x, y$ , and $z$ are integers that satisfy $0 \le x \le 2, 0 \le y \le 3$ , and $0 \le z \le 4$ . Two distinct points are randomly chosen from $S$ . The probability that the midpoint of the segment they determine also belongs to $S$ is $m/n$ , where $m$ and $n$ are relatively prime positive integers. Find $m + n$ . Answer:

Question <b>11</b> Not yet answered Points out of 5	In a rectangular array of points, with 5 rows and $N$ columns, the points are numbered consecutively from left to right beginning with the top row. Thus the top row is numbered 1 through $N$ , the second row is numbered $N + 1$ through $2N$ , and so forth. Five points, $P_1, P_2, P_3, P_4$ , and $P_5$ , are selected so that each $P_i$ is in row $i$ . Let $x_i$ be the number associated with $P_i$ . Now renumber the array consecutively from top to bottom, beginning with the first column. Let $y_i$ be the number associated with $P_i$ after the renumbering. It is found that $x_1 = y_2, x_2 = y_1, x_3 = y_4, x_4 = y_5$ , and $x_5 = y_3$ . Find the smallest possible value of $N$ .
Question 12 Not yet answered Points out of 5	A sphere is inscribed in the tetrahedron whose vertices are $A = (6, 0, 0), B = (0, 4, 0), C = (0, 0, 2)$ , and $D = (0, 0, 0)$ . The radius of the sphere is $m/n$ , where $m$ and $n$ are relatively prime positive integers. Find $m + n$ .
Question 13 Not yet answered Points out of 5	In a certain circle, the chord of a $d$ -degree arc is 22 centimeters long, and the chord of a $2d$ -degree arc is 20 centimeters longer than the chord of a $3d$ -degree arc, where $d < 120$ . The length of the chord of a $3d$ -degree arc is $-m + \sqrt{n}$ centimeters, where $m$ and $n$ are positive integers. Find $m + n$ .
Question <b>14</b> Not yet answered Points out of 5	A mail carrier delivers mail to the nineteen houses on the east side of Elm Street. The carrier notices that no two adjacent houses ever get mail on the same day, but that there are never more than two houses in a row that get no mail on the same day. How many different patterns of mail delivery are possible?
Question <b>15</b> Not yet answered Points out of 5	The numbers $1, 2, 3, 4, 5, 6, 7$ , and $8$ are randomly written on the faces of a regular octahedron so that each face contains a different number. The probability that no two consecutive numbers, where $8$ and $1$ are considered to be consecutive, are written on faces that share an edge is $m/n$ , where $m$ and $n$ are relatively prime positive integers. Find $m + n$ .