2001 AIME II

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Question 1

Not yet answered

Points out of 5

Let N be the largest positive integer with the following property: reading from left to right, each pair of consecutive digits of N forms a perfect square. What are the leftmost three digits of N?

Answer:

Question 2

Not yet answered

Points out of 5

Each of the 2001 students at a high school studies either Spanish or French, and some study both. The number who study Spanish is between 80 percent and 85 percent of the school population, and the number who study French is between 30 percent and 40 percent. Let m be the smallest number of students who could study both languages, and let M be the largest number of students who could study both languages. Find $M - m$.

Answer:

Question 3

Not yet answered

Points out of 5

Given that

$$\begin{aligned} x_1 &= 211, \\ x_2 &= 375, \\ x_3 &= 420, \\ x_4 &= 523, \text{ and} \\ x_n &= x_{n-1} - x_{n-2} + x_{n-3} - x_{n-4} \text{ when } n \geq 5, \end{aligned}$$

find the value of $x_{531} + x_{753} + x_{975}$.

Answer:

Question 4

Not yet answered

Points out of 5

Let $R = (8, 6)$. The lines whose equations are $8y = 15x$ and $10y = 3x$ contain points P and Q, respectively, such that R is the midpoint of $\overline{PQ}$. The length of PQ equals $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 5

Not yet answered

Points out of 5

A set of positive numbers has the *triangle property* if it has three distinct elements that are the lengths of the sides of a triangle whose area is positive. Consider sets $\{4, 5, 6, \ldots, n\}$ of consecutive positive integers, all of whose ten-element subsets have the triangle property. What is the largest possible value of n?

Answer:

Question 6

Not yet answered

Points out of 5

Square $ABCD$ is inscribed in a circle. Square $EFGH$ has vertices E and F on $\overline{CD}$ and vertices G and H on the circle. The ratio of the area of square $EFGH$ to the area of square $ABCD$ can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers and $m < n$. Find $10n + m$.

Answer:

Question 7

Not yet answered

Points out of 5

Let $\triangle PQR$ be a right triangle with $PQ = 90$, $PR = 120$, and $QR = 150$. Let C_1 be the inscribed circle. Construct $\overline{ST}$ with S on $\overline{PR}$ and T on $\overline{QR}$, such that $\overline{ST}$ is perpendicular to $\overline{PR}$ and tangent to C_1. Construct $\overline{UV}$ with U on $\overline{PQ}$ and V on $\overline{QR}$ such that $\overline{UV}$ is perpendicular to $\overline{PQ}$ and tangent to C_1. Let C_2 be the inscribed circle of $\triangle RST$ and C_3 the inscribed circle of $\triangle QUV$. The distance between the centers of C_2 and C_3 can be written as $\sqrt{10n}$. What is n?

Answer:

Question 8

Not yet answered

Points out of 5

A certain function f has the properties that $f(3x) = 3f(x)$ for all positive real values of x, and that $f(x) = 1 - |x - 2|$ for $1 \leq x \leq 3$. Find the smallest x for which $f(x) = f(2001)$.

Answer:

Question 9

Not yet answered

Points out of 5

Each unit square of a 3-by-3 unit-square grid is to be colored either blue or red. For each square, either color is equally likely to be used. The probability of obtaining a grid that does not have a 2-by-2 red square is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 10

Not yet answered

Points out of 5

How many positive integer multiples of 1001 can be expressed in the form $10^j - 10^i$, where i and j are integers and $0 \leq i < j \leq 99$?

Answer:

Question 11

Not yet answered

Points out of 5

Club Truncator is in a soccer league with six other teams, each of which it plays once. In any of its 6 matches, the probabilities that Club Truncator will win, lose, or tie are each $\frac{1}{3}$. The probability that Club Truncator will finish the season with more wins than losses is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Answer:

Question 12

Not yet answered

Points out of 5

Given a triangle, its midpoint triangle is obtained by joining the midpoints of its sides. A sequence of polyhedra P_i is defined recursively as follows: P_0 is a regular tetrahedron whose volume is 1. To obtain P_{i+1}, replace the midpoint triangle of every face of P_i by an outward-pointing regular tetrahedron that has the midpoint triangle as a face. The volume of P_3 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Answer:

Question 13

Not yet answered

Points out of 5

In quadrilateral $ABCD$, $\angle BAD \cong \angle ADC$ and $\angle ABD \cong \angle BCD$, $AB = 8$, $BD = 10$, and $BC = 6$. The length CD may be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Answer:

Question 14

Not yet answered

Points out of 5

There are $2n$ complex numbers that satisfy both $z^{28} - z^8 - 1 = 0$ and $|z| = 1$. These numbers have the form $z_m = \cos\theta_m + i\sin\theta_m$, where $0 \leq \theta_1 < \theta_2 < \ldots < \theta_{2n} < 360$ and angles are measured in degrees. Find the value of $\theta_2 + \theta_4 + \ldots + \theta_{2n}$.

Answer:

Question **15**

Not yet answered

Points out of 5

Let $EFGH$, $EFDC$, and $EHBC$ be three adjacent square faces of a cube, for which $EC = 8$, and let A be the eighth vertex of the cube. Let I, J, and K, be the points on $\overline{EF}$, $\overline{EH}$, and $\overline{EC}$, respectively, so that $EI = EJ = EK = 2$. A solid S is obtained by drilling a tunnel through the cube. The sides of the tunnel are planes parallel to $\overline{AE}$, and containing the edges, $\overline{IJ}$, $\overline{JK}$, and $\overline{KI}$. The surface area of S, including the walls of the tunnel, is $m + n\sqrt{p}$, where m, n, and p are positive integers and p is not divisible by the square of any prime. Find $m + n + p$.

Answer: