2002 AIME II

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Question 1

Not yet answered

Points out of 5

Given that

(1) x and y are both integers between 100 and 999, inclusive,

(2) y is the number formed by reversing the digits of x, and

(3) $z = |x - y|$.

How many distinct values of z are possible?

Answer:

Question 2

Not yet answered

Points out of 5

Three vertices of a cube are $P = (7, 12, 10)$, $Q = (8, 8, 1)$, and $R = (11, 3, 9)$. What is the surface area of the cube?

Answer:

Question 3

Not yet answered

Points out of 5

It is given that $\log_6 a + \log_6 b + \log_6 c = 6$, where a, b, and c are positive integers that form an increasing geometric sequence and $b - a$ is the square of an integer. Find $a + b + c$.

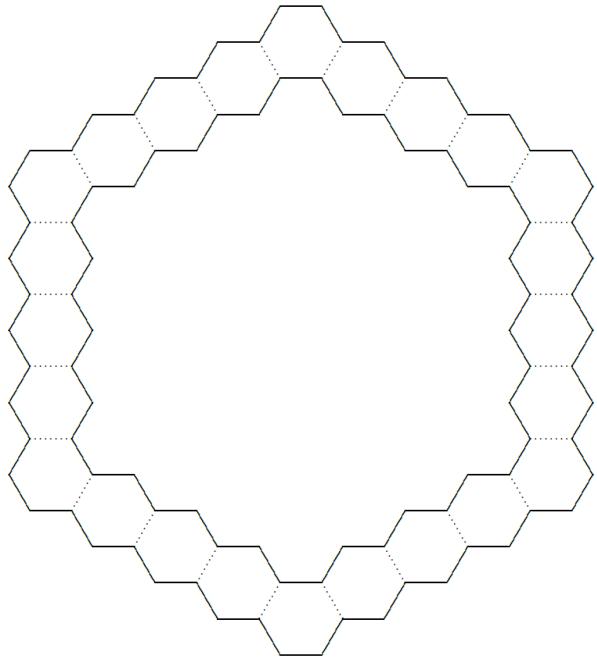
Answer:

Question 4

Not yet answered

Points out of 5

Patio blocks that are hexagons 1 unit on a side are used to outline a garden by placing the blocks edge to edge with n on each side. The diagram indicates the path of blocks around the garden when $n = 5$. If $n = 202$, then the area of the garden enclosed by the path, not including the path itself, is $m\left(\sqrt{3}/2\right)$ square units, where m is a positive integer.



Find the remainder when m is divided by 1000.

Answer:

Question 5

Not yet answered

Points out of 5

Find the sum of all positive integers $a = 2^n 3^m$ where n and m are non-negative integers, for which a^6 is not a divisor of 6^a

Answer:

Question 6

Not yet answered

Points out of 5

Find the integer that is closest to $1000 \sum_{n=3}^{10000} \frac{1}{n^2 - 4}$.

Answer:

Question 7

Not yet answered

Points out of 5

It is known that, for all positive integers k,

$$1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Find the smallest positive integer k such that $1^2 + 2^2 + 3^2 + \ldots + k^2$ is a multiple of 200.

Answer:

Question 8

Not yet answered

Points out of 5

Find the least positive integer k for which the equation $\left\lfloor \frac{2002}{n} \right\rfloor = k$ has no integer solutions for n. (The notation $\lfloor x \rfloor$ means the greatest integer less than or equal to x.)

Answer:

Question 9

Not yet answered

Points out of 5

Let $\mathcal{S}$ be the set $\{1, 2, 3, \ldots, 10\}$ Let n be the number of sets of two non-empty disjoint subsets of $\mathcal{S}$. (Disjoint sets are defined as sets that have no common elements.) Find the remainder obtained when n is divided by 1000.

Answer:

Question 10

Not yet answered

Points out of 5

While finding the sine of a certain angle, an absent-minded professor failed to notice that his calculator was not in the correct angular mode. He was lucky to get the right answer. The two least positive real values of x for which the sine of x degrees is the same as the sine of x radians are $\dfrac{m\pi}{n-\pi}$ and $\dfrac{p\pi}{q+\pi}$, where m, n, p, and q are positive integers. Find $m+n+p+q$.

Answer:

Question 11

Not yet answered

Points out of 5

Two distinct, real, infinite geometric series each have a sum of 1 and have the same second term. The third term of one of the series is $1/8$, and the second term of both series can be written in the form $\frac{\sqrt{m}-n}{p}$, where m, n, and p are positive integers and m is not divisible by the square of any prime. Find $100m + 10n + p$.

Answer:

Question 12

Not yet answered

Points out of 5

A basketball player has a constant probability of $.4$ of making any given shot, independent of previous shots. Let a_n be the ratio of shots made to shots attempted after n shots. The probability that $a_{10} = .4$ and $a_n \leq .4$ for all n such that $1 \leq n \leq 9$ is given to be $p^a q^b r / (s^c)$ where p, q, r, and s are primes, and a, b, and c are positive integers. Find $(p+q+r+s)(a+b+c)$.

Answer:

Question 13

Not yet answered

Points out of 5

In triangle ABC, point D is on $\overline{BC}$ with $CD = 2$ and $DB = 5$, point E is on $\overline{AC}$ with $CE = 1$ and $EA = 3$, $AB = 8$, and $\overline{AD}$ and $\overline{BE}$ intersect at P. Points Q and R lie on $\overline{AB}$ so that $\overline{PQ}$ is parallel to $\overline{CA}$ and $\overline{PR}$ is parallel to $\overline{CB}$. It is given that the ratio of the area of triangle PQR to the area of triangle ABC is m/n, where m and n are relatively prime positive integers. Find $m+n$.

Answer:

Question 14

Not yet answered

Points out of 5

The perimeter of triangle APM is 152, and the angle PAM is a right angle. A circle of radius 19 with center O on $\overline{AP}$ is drawn so that it is tangent to $\overline{AM}$ and $\overline{PM}$. Given that $OP = m/n$ where m and n are relatively prime positive integers, find $m+n$.

Answer:

Question 15

Not yet answered

Points out of 5

Circles $\mathcal{C}_1$ and $\mathcal{C}_2$ intersect at two points, one of which is $(9,6)$, and the product of the radii is 68. The x-axis and the line $y = mx$, where $m > 0$, are tangent to both circles. It is given that m can be written in the form $a\sqrt{b}/c$, where a, b, and c are positive integers, b is not divisible by the square of any prime, and a and c are relatively prime. Find $a+b+c$.

Answer: