2005 AIME I

For more practice and resources, visit ziml.areteem.org

Question **1**

Not yet answered

Points out of 5

Six congruent circles form a ring with each circle externally tangent to two circles adjacent to it. All circles are internally tangent to a circle C with radius 30. Let K be the area of the region inside circle C and outside of the six circles in the ring. Find $\lfloor K \rfloor$ (the floor function).

Answer:

Question **2**

Not yet answered

Points out of 5

For each positive integer k, let S_k denote the increasing arithmetic sequence of integers whose first term is 1 and whose common difference is k. For example, S_3 is the sequence $1, 4, 7, 10, \ldots$. For how many values of k does S_k contain the term 2005?

Answer:

Question **3**

Not yet answered

Points out of 5

How many positive integers have exactly three proper divisors (positive integral divisors excluding itself), each of which is less than 50?

Answer:

Question **4**

Not yet answered

Points out of 5

The director of a marching band wishes to place the members into a formation that includes all of them and has no unfilled positions. If they are arranged in a square formation, there are 5 members left over. The director realizes that if he arranges the group in a formation with 7 more rows than columns, there are no members left over. Find the maximum number of members this band can have.

Answer:

Question **5**

Not yet answered

Points out of 5

Robert has 4 indistinguishable gold coins and 4 indistinguishable silver coins. Each coin has an engraving of one face on one side, but not on the other. He wants to stack the eight coins on a table into a single stack so that no two adjacent coins are face to face. Find the number of possible distinguishable arrangements of the 8 coins.

Answer:

Question 6

Not yet answered

Points out of 5

Let P be the product of the nonreal roots of $x^4 - 4x^3 + 6x^2 - 4x = 2005$. Find $\lfloor P \rfloor$.

Answer:

Question 7

Not yet answered

Points out of 5

In quadrilateral $ABCD,\ BC = 8,\ CD = 12,\ AD = 10$, and $m\angle A = m\angle B = 60^\circ$. Given that $AB = p + \sqrt{q}$, where p and q are positive integers, find $p + q$.

Answer:

Question 8

Not yet answered

Points out of 5

The equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$ has three real roots. Given that their sum is $\frac{m}{n}$ where m and n are relatively prime positive integers, find $m + n$.

Answer:

Question 9

Not yet answered

Points out of 5

Twenty seven unit cubes are painted orange on a set of four faces so that two non-painted faces share an edge. The 27 cubes are randomly arranged to form a $3 \times 3 \times 3$ cube. Given the probability of the entire surface area of the larger cube is orange is $\frac{p^a}{q^b r^c}$, where p, q, and r are distinct primes and a, b, and c are positive integers, find $a + b + c + p + q + r$.

Answer:

Question 10

Not yet answered

Points out of 5

Triangle ABC lies in the cartesian plane and has an area of 70. The coordinates of B and C are $(12, 19)$ and $(23, 20)$, respectively, and the coordinates of A are (p, q). The line containing the median to side BC has slope -5. Find the largest possible value of $p + q$.

Answer:

Question 11

Not yet answered

Points out of 5

A semicircle with diameter d is contained in a square whose sides have length 8. Given the maximum value of d is $m - \sqrt{n}$, find $m + n$.

Answer:

Question **12**

Not yet answered

Points out of 5

For positive integers n, let $\tau(n)$ denote the number of positive integer divisors of n, including 1 and n. For example, $\tau(1) = 1$ and $\tau(6) = 4$. Define $S(n)$ by $S(n) = \tau(1) + \tau(2) + \cdots + \tau(n)$. Let a denote the number of positive integers $n \leq 2005$ with $S(n)$ odd, and let b denote the number of positive integers $n \leq 2005$ with $S(n)$ even. Find $|a - b|$.

Answer:

Question **13**

Not yet answered

Points out of 5

A particle moves in the Cartesian plane according to the following rules:

1. From any lattice point (a, b), the particle may only move to $(a + 1, b)$, $(a, b + 1)$, or $(a + 1, b + 1)$.

2. There are no right angle turns in the particle's path.

How many different paths can the particle take from $(0, 0)$ to $(5, 5)$?

Answer:

Question **14**

Not yet answered

Points out of 5

Consider the points $A(0, 12), B(10, 9), C(8, 0)$, and $D(-4, 7)$. There is a unique square S such that each of the four points is on a different side of S. Let K be the area of S. Find the remainder when $10K$ is divided by 1000.

Answer:

Question **15**

Not yet answered

Points out of 5

Triangle ABC has $BC = 20$. The incircle of the triangle evenly trisects the median AD. If the area of the triangle is $m\sqrt{n}$ where m and n are integers and n is not divisible by the square of a prime, find $m + n$.

Answer: