2005 AIME II

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Question 1

Not yet answered

Points out of 5

A game uses a deck of n different cards, where n is an integer and $n \geq 6$. The number of possible sets of 6 cards that can be drawn from the deck is 6 times the number of possible sets of 3 cards that can be drawn. Find n.

Answer:

Question 2

Not yet answered

Points out of 5

A hotel packed breakfast for each of three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese, and fruit rolls. The preparer wrapped each of the nine rolls and once wrapped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. Given that the probability each guest got one roll of each type is $\frac{m}{n}$, where m and n are relatively prime integers, find $m + n$.

Answer:

Question 3

Not yet answered

Points out of 5

An infinite geometric series has sum 2005. A new series, obtained by squaring each term of the original series, has 10 times the sum of the original series. The common ratio of the original series is $\frac{m}{n}$ where m and n are relatively prime integers. Find $m + n$.

Answer:

Question 4

Not yet answered

Points out of 5

Find the number of positive integers that are divisors of at least one of $10^{10}, 15^7, 18^{11}$.

Answer:

Question 5

Not yet answered

Points out of 5

Determine the number of ordered pairs (a, b) of integers such that $\log_a b + 6 \log_b a = 5, 2 \leq a \leq 2005$, and $2 \leq b \leq 2005$.

Answer:

Question 6

Not yet answered

Points out of 5

The cards in a stack of $2n$ cards are numbered consecutively from 1 through $2n$ from top to bottom. The top n cards are removed, kept in order, and form pile A. The remaining cards form pile B. The cards are then restacked by taking cards alternately from the tops of pile B and A, respectively. In this process, card number $(n+1)$ becomes the bottom card of the new stack, card number 1 is on top of this card, and so on, until piles A and B are exhausted. If, after the restacking process, at least one card from each pile occupies the same position that it occupied in the original stack, the stack is named magical. Find the number of cards in the magical stack in which card number 131 retains its original position.

Answer:

Question 7

Not yet answered

Points out of 5

Let $x = \dfrac{4}{(\sqrt{5}+1)(\sqrt[4]{5}+1)(\sqrt[8]{5}+1)(\sqrt[16]{5}+1)}$. Find $(x+1)^{48}$.

Answer:

Question 8

Not yet answered

Points out of 5

Circles C_1 and C_2 are externally tangent, and they are both internally tangent to circle C_3. The radii of C_1 and C_2 are 4 and 10, respectively, and the centers of the three circles are all collinear. A chord of C_3 is also a common external tangent of C_1 and C_2. Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where $m, n,$ and p are positive integers, m and p are relatively prime, and n is not divisible by the square of any prime, find $m+n+p$.

Answer:

Question 9

Not yet answered

Points out of 5

For how many positive integers n less than or equal to 1000 is $(\sin t + i\cos t)^n = \sin nt + i\cos nt$ true for all real t?

Answer:

Question 10

Not yet answered

Points out of 5

Given that O is a regular octahedron, that C is the cube whose vertices are the centers of the faces of O, and that the ratio of the volume of O to that of C is $\frac{m}{n}$, where m and n are relatively prime integers, find $m+n$.

Answer:

Question 11

Not yet answered

Points out of 5

Let m be a positive integer, and let $a_0, a_1, \ldots, a_m$ be a sequence of integers such that $a_0 = 37, a_1 = 72, a_m = 0$, and $a_{k+1} = a_{k-1} - \frac{3}{a_k}$ for $k = 1, 2, \ldots, m-1$. Find m.

Note: Clearly, the stipulation that the sequence is composed of integers is a minor oversight, as the term a_2, for example, is obviously not integral.

Answer:

Question 12

Not yet answered

Points out of 5

Square $ABCD$ has center O, $AB = 900$, E and F are on AB with $AE < BF$ and E between A and F, $m\angle EOF = 45^\circ$, and $EF = 400$. Given that $BF = p + q\sqrt{r}$, where $p, q,$ and r are positive integers and r is not divisible by the square of any prime, find $p + q + r$.

Answer:

Question 13

Not yet answered

Points out of 5

Let $P(x)$ be a polynomial with integer coefficients that satisfies $P(17) = 10$ and $P(24) = 17$. Given that $P(n) = n + 3$ has two distinct integer solutions n_1 and n_2, find the product $n_1 \cdot n_2$.

Answer:

Question 14

Not yet answered

Points out of 5

In triangle $ABC, AB = 13, BC = 15,$ and $CA = 14$. Point D is on $\overline{BC}$ with $CD = 6$. Point E is on $\overline{BC}$ such that $\angle BAE \cong \angle CAD$. Given that $BE = \frac{p}{q}$ where p and q are relatively prime positive integers, find q.

Answer:

Question 15

Not yet answered

Points out of 5

Let w_1 and w_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$, respectively. Let m be the smallest positive value of a for which the line $y = ax$ contains the center of a circle that is externally tangent to w_2 and internally tangent to w_1. Given that $m^2 = \frac{p}{q}$, where p and q are relatively prime integers, find $p + q$.

Answer: