2007 AIME I

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Question 1

Not yet answered

Points out of 5

How many positive perfect squares less than 10^6 are multiples of 24?

Answer:

Question 2

Not yet answered

Points out of 5

A 100 foot long moving walkway moves at a constant rate of 6 feet per second. Al steps onto the start of the walkway and stands. Bob steps onto the start of the walkway two seconds later and strolls forward along the walkway at a constant rate of 4 feet per second. Two seconds after that, Cy reaches the start of the walkway and walks briskly forward beside the walkway at a constant rate of 8 feet per second. At a certain time, one of these three persons is exactly halfway between the other two. At that time, find the distance in feet between the start of the walkway and the middle person.

Answer:

Question 3

Not yet answered

Points out of 5

The complex number z is equal to $9 + bi$, where b is a positive real number and $i^2 = -1.$ Given that the imaginary parts of z^2 and z^3 are the same, what is b equal to?

Answer:

Question 4

Not yet answered

Points out of 5

Three planets orbit a star circularly in the same plane. Each moves in the same direction and moves at constant speed. Their periods are 60, 84, and 140. The three planets and the star are currently collinear. What is the fewest number of years from now that they will all be collinear again?

Answer:

Question 5

Not yet answered

Points out of 5

The formula for converting a Fahrenheit temperature F to the corresponding Celsius temperature C is $C = \frac{5}{9}(F - 32).$ An integer Fahrenheit temperature is converted to Celsius, rounded to the nearest integer, converted back to Fahrenheit, and again rounded to the nearest integer.

For how many integer Fahrenheit temperatures between 32 and 1000 inclusive does the original temperature equal the final temperature?

Answer:

Question 6

Not yet answered

Points out of 5

A frog is placed at the origin on the number line, and moves according to the following rule: in a given move, the frog advances to either the closest point with a greater integer coordinate that is a multiple of 3, or to the closest point with a greater integer coordinate that is a multiple of 13. A *move sequence* is a sequence of coordinates which correspond to valid moves, beginning with 0, and ending with 39. For example, $0,\ 3,\ 6,\ 13,\ 15,\ 26,\ 39$ is a move sequence. How many move sequences are possible for the frog?

Answer:

Question 7

Not yet answered

Points out of 5

Let $N = \sum_{k=1}^{1000} k(\lceil \log_{\sqrt{2}} k \rceil - \lfloor \log_{\sqrt{2}} k \rfloor)$

Find the remainder when N is divided by 1000. ($\lfloor k \rfloor$ is the greatest integer less than or equal to k, and $\lceil k \rceil$ is the least integer greater than or equal to k.)

Answer:

Question 8

Not yet answered

Points out of 5

The polynomial $P(x)$ is cubic. What is the largest value of k for which the polynomials $Q_1(x) = x^2 + (k-29)x - k$ and $Q_2(x) = 2x^2 + (2k-43)x + k$ are both factors of $P(x)$?

Answer:

Question 9

Not yet answered

Points out of 5

In right triangle ABC with right angle C, $CA = 30$ and $CB = 16$. Its legs CA and CB are extended beyond A and B. Points O_1 and O_2 lie in the exterior of the triangle and are the centers of two circles with equal radii. The circle with center O_1 is tangent to the hypotenuse and to the extension of leg CA, the circle with center O_2 is tangent to the hypotenuse and to the extension of leg CB, and the circles are externally tangent to each other. The length of the radius of either circle can be expressed as p/q, where p and q are relatively prime positive integers. Find $p + q$.

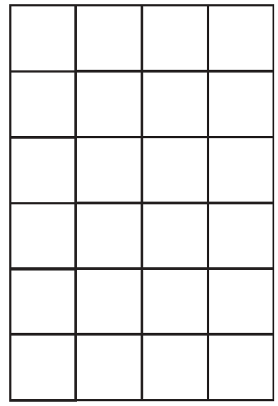
Answer:

Question **10**

Not yet answered

Points out of 5

In a 6 x 4 grid (6 rows, 4 columns), 12 of the 24 squares are to be shaded so that there are two shaded squares in each row and three shaded squares in each column. Let N be the number of shadings with this property. Find the remainder when N is divided by 1000.



Answer:

Question **11**

Not yet answered

Points out of 5

For each positive integer p, let $b(p)$ denote the unique positive integer k such that $|k - \sqrt{p}| < \frac{1}{2}$. For example, $b(6) = 2$ and $b(23) = 5$. If $S = \sum_{p=1}^{2007} b(p)$, find the remainder when S is divided by 1000.

Answer:

Question **12**

Not yet answered

Points out of 5

In isosceles triangle $\triangle ABC$, A is located at the origin and B is located at $(20, 0)$. Point C is in the first quadrant with $AC = BC$ and angle $BAC = 75^{\circ}$. If triangle ABC is rotated counterclockwise about point A until the image of C lies on the positive y-axis, the area of the region common to the original and the rotated triangle is in the form $p\sqrt{2} + q\sqrt{3} + r\sqrt{6} + s$, where p, q, r, s are integers. Find $\dfrac{p-q+r-s}{2}$.

Answer:

Question **13**

Not yet answered

Points out of 5

A square pyramid with base $ABCD$ and vertex E has eight edges of length 4. A plane passes through the midpoints of AE, BC, and CD. The plane's intersection with the pyramid has an area that can be expressed as $\sqrt{p}$. Find p.

Answer:

Question **14**

Not yet answered

Points out of 5

A sequence is defined over non-negative integral indexes in the following way: $a_0 = a_1 = 3$, $a_{n+1}a_{n-1} = a_n^2 + 2007$.

Find the greatest integer that does not exceed $\dfrac{a_{2006}^2 + a_{2007}^2}{a_{2006}a_{2007}}$

Answer:

Question **15**

Not yet answered

Points out of 5

Let ABC be an equilateral triangle, and let D and F be points on sides BC and AB, respectively, with $FA = 5$ and $CD = 2$. Point E lies on side CA such that angle $DEF = 60^\circ$. The area of triangle DEF is $14\sqrt{3}$. The two possible values of the length of side AB are $p \pm q\sqrt{r}$, where p and q are rational, and r is an integer not divisible by the square of a prime. Find r.

Answer: