2010 AIME II

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Question 1

Not yet answered

Points out of 5

Let N be the greatest integer multiple of 36 all of whose digits are even and no two of whose digits are the same. Find the remainder when N is divided by 1000.

Answer:

Question 2

Not yet answered

Points out of 5

A point P is chosen at random in the interior of a unit square S. Let $d(P)$ denote the distance from P to the closest side of S. The probability that $\frac{1}{5} \leq d(P) \leq \frac{1}{3}$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 3

Not yet answered

Points out of 5

Let K be the product of all factors $(b - a)$ (not necessarily distinct) where a and b are integers satisfying $1 \leq a < b \leq 20$. Find the greatest positive integer n such that 2^n divides K.

Answer:

Question 4

Not yet answered

Points out of 5

Dave arrives at an airport which has twelve gates arranged in a straight line with exactly 100 feet between adjacent gates. His departure gate is assigned at random. After waiting at that gate, Dave is told the departure gate has been changed to a different gate, again at random. Let the probability that Dave walks 400 feet or less to the new gate be a fraction $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 5

Not yet answered

Points out of 5

Positive numbers x, y, and z satisfy $xyz = 10^{81}$ and $(\log_{10} x)(\log_{10} yz) + (\log_{10} y)(\log_{10} z) = 468$. Find $\sqrt{(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2}$.

Answer:

Question 6

Not yet answered

Points out of 5

Find the smallest positive integer n with the property that the polynomial $x^4 - nx + 63$ can be written as a product of two nonconstant polynomials with integer coefficients.

Answer:

Question 7

Not yet answered

Points out of 5

Let $P(z) = z^3 + az^2 + bz + c$, where a, b, and c are real. There exists a complex number w such that the three roots of $P(z)$ are $w + 3i$, $w + 9i$, and $2w - 4$, where $i^2 = -1$. Find $|a + b + c|$.

Answer:

Question 8

Not yet answered

Points out of 5

Let N be the number of ordered pairs of nonempty sets $\mathcal{A}$ and $\mathcal{B}$ that have the following properties:

(i) $\mathcal{A} \cup \mathcal{B} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$,

(ii) $\mathcal{A} \cap \mathcal{B} = \emptyset$,

(iii) The number of elements of $\mathcal{A}$ is not an element of $\mathcal{A}$, and

(iv) The number of elements of $\mathcal{B}$ is not an element of $\mathcal{B}$.

Find N.

Answer:

Question 9

Not yet answered

Points out of 5

Let $ABCDEF$ be a regular hexagon. Let G, H, I, J, K, and L be the midpoints of sides AB, BC, CD, DE, EF, and AF, respectively. The segments $\overline{AH}$, $\overline{BI}$, $\overline{CJ}$, $\overline{DK}$, $\overline{EL}$, and $\overline{FG}$ bound a smaller regular hexagon. Let the ratio of the area of the smaller hexagon to the area of $ABCDEF$ be expressed as a fraction $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 10

Not yet answered

Points out of 5

Find the number of second-degree polynomials $f(x)$ with integer coefficients and integer zeros for which $f(0) = 2010$.

Answer:

Question 11

Not yet answered

Points out of 5

Define a *T-grid* to be a 3×3 matrix which satisfies the following two properties:

(1) Exactly five of the entries are 1's, and the remaining four entries are 0's.

(2) Among the eight rows, columns, and long diagonals (the long diagonals are $\{a_{13}, a_{22}, a_{31}\}$ and $\{a_{11}, a_{22}, a_{33}\}$, no more than one of the eight has all three entries equal.

Find the number of distinct *T-grids*.

Answer:

Question 12

Not yet answered

Points out of 5

Two noncongruent integer-sided isosceles triangles have the same perimeter and the same area. The ratio of the lengths of the bases of the two triangles is $8 : 7$. Find the minimum possible value of their common perimeter.

Answer:

Question 13

Not yet answered

Points out of 5

The 52 cards in a deck are numbered $1, 2, \cdots, 52$. Alex, Blair, Corey, and Dylan each picks a card from the deck without replacement and with each card being equally likely to be picked, The two persons with lower numbered cards from a team, and the two persons with higher numbered cards form another team. Let $p(a)$ be the probability that Alex and Dylan are on the same team, given that Alex picks one of the cards a and $a + 9$, and Dylan picks the other of these two cards. The minimum value of $p(a)$ for which $p(a) \geq \frac{1}{2}$ can be written as $\frac{m}{n}$. where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 14

Not yet answered

Points out of 5

Triangle ABC with right angle at C, $\angle BAC < 45^\circ$ and $AB = 4$. Point P on $\overline{AB}$ is chosen such that $\angle APC = 2\angle ACP$ and $CP = 1$. The ratio $\frac{AP}{BP}$ can be represented in the form $p + q\sqrt{r}$, where p, q, r are positive integers and r is not divisible by the square of any prime. Find $p + q + r$.

Answer:

Question **15**

Not yet answered

Points out of 5

In triangle ABC, $AC = 13$, $BC = 14$, and $AB = 15$. Points M and D lie on AC with $AM = MC$ and $\angle ABD = \angle DBC$. Points N and E lie on AB with $AN = NB$ and $\angle ACE = \angle ECB$. Let P be the point, other than A, of intersection of the circumcircles of $\triangle AMN$ and $\triangle ADE$. Ray AP meets BC at Q. The ratio $\frac{BQ}{CQ}$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m - n$.

Answer: