

# 2017 AIME I 

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Question 1
Not yet answered

Points out of 5

## Question 2

Not yet answered
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## Question 3

Not yet answered

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## Question 4

Not yet answered
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## Question 5

Not yet answered
Points out of 5

Fifteen distinct points are designated on $\triangle A B C$ : the 3 vertices $A, B$, and $C ; 3$ other points on side $\overline{A B} ; 4$ other points on side $\overline{B C}$; and 5 other points on side $\overline{C A}$. Find the number of triangles with positive area whose vertices are among these 15 points.

## Answer:

When each of 702,787 , and 855 is divided by the positive integer $m$, the remainder is always the positive integer $r$. When each of 412,722 , and 815 is divided by the positive integer $n$, the remainder is always the positive integer $s \neq r$. Find $m+n+r+s$.

## Answer:

For a positive integer $n$, let $d_{n}$ be the units digit of $1+2+\cdots+n$. Find the remainder when

$$
\sum_{n=1}^{2017} d_{n}
$$

is divided by 1000 .

Answer:

A pyramid has a triangular base with side lengths 20,20 , and 24 . The three edges of the pyramid from the three corners of the base to the fourth vertex of the pyramid all have length 25 . The volume of the pyramid is $m \sqrt{ } \bar{n}$, where $m$ and $n$ are positive integers, and $n$ is not divisible by the square of any prime. Find $m+n$.

Answer:

A rational number written in base eight is $\underline{a b}$. $\underline{c d}$, where all digits are nonzero. The same number in base twelve is $\underline{b b} . \underline{b a}$. Find the base-ten number $\underline{a b c}$.

## Answer:

$\square$

## Question 6

Not yet answered

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## Question 7

Not yet answered
Points out of 5

## Question 8

Not yet answered
Points out of 5

## Question 9

Not yet answered
Points out of 5

A circle is circumscribed around an isosceles triangle whose two congruent angles have degree measure $x$. Two points are chosen independently and uniformly at random on the circle, and a chord is drawn between them. The probability that the chord intersects the triangle is $\frac{14}{25}$. Find the difference between the largest and smallest possible values of $x$.

## Answer:

For nonnegative integers $a$ and $b$ with $a+b \leq 6$, let $T(a, b)=\binom{6}{a}\binom{6}{b}\binom{6}{a+b}$. Let $S$ denote the sum of all $T(a, b)$, where $a$ and $b$ are nonnegative integers with $a+b \leq 6$. Find the remainder when $S$ is divided by 1000 .

## Answer:

Two real numbers $a$ and $b$ are chosen independently and uniformly at random from the interval $(0,75)$. Let $O$ and $P$ be two points on the plane with $O P=200$. Let $Q$ and $R$ be on the same side of line $O P$ such that the degree measures of $\angle P O Q$ and $\angle P O R$ are $a$ and $b$ respectively, and $\angle O Q P$ and $\angle O R P$ are both right angles. The probability that $Q R \leq 100$ is equal to $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m+n$.

## Answer:

Let $a_{10}=10$, and for each integer $n>10$ let $a_{n}=100 a_{n-1}+n$. Find the least $n>10$ such that $a_{n}$ is a multiple of 99 .

## Answer:

Let $z_{1}=18+83 i, z_{2}=18+39 i$, and $z_{3}=78+99 i$, where $i=\sqrt{-1}$. Let $z$ be the unique complex number with the properties that $\frac{z_{3}-z_{1}}{z_{2}-z_{1}} \cdot \frac{z-z_{2}}{z-z_{3}}$ is a real number and the imaginary part of $z$ is the greatest possible. Find the real part of $z$.

Answer: $\square$

## Question 11

Not yet answered
Points out of 5

## Question 12

Not yet answered
Points out of 5

## Question 13

Not yet answered
Points out of 5

## Question 14

Not yet answered
Points out of 5

Consider arrangements of the 9 numbers $1,2,3, \ldots, 9$ in a $3 \times 3$ array. For each such arrangement, let $a_{1}, a_{2}$, and $a_{3}$ be the medians of the numbers in rows 1,2 , and 3 respectively, and let $m$ be the median of $\left\{a_{1}, a_{2}, a_{3}\right\}$. Let $Q$ be the number of arrangements for which $m=5$. Find the remainder when $Q$ is divided by 1000 .

## Answer:

$\square$

Call a set $S$ product-free if there do not exist $a, b, c \in S$ (not necessarily distinct) such that $a b=c$. For example, the empty set and the set $\{16,20\}$ are product-free, whereas the sets $\{4,16\}$ and $\{2,8,16\}$ are not product-free. Find the number of product-free subsets of the set $\{1,2,3,4,5,6,7,8,9,10\}$.

## Answer:

$\square$

For every $m \geq 2$, let $Q(m)$ be the least positive integer with the following property: For every $n \geq Q(m)$, there is always a perfect cube $k^{3}$ in the range $n<k^{3} \leq m \cdot n$. Find the remainder when

$$
\sum_{m=2}^{2017} Q(m)
$$

is divided by 1000 .

## Answer:

Let $a>1$ and $x>1$ satisfy $\log _{a}\left(\log _{a}\left(\log _{a} 2\right)+\log _{a} 24-128\right)=128$ and $\log _{a}\left(\log _{a} x\right)=256$. Find the remainder when $x$ is divided by 1000 .

Answer: $\square$

Question 15
Not yet answered
Points out of 5

The area of the smallest equilateral triangle with one vertex on each of the sides of the right triangle with side lengths $2 \sqrt{3}, 5$, and $\sqrt{37}$, as shown, is $\frac{m \sqrt{p}}{n}$, where $m, n$, and $p$ are positive integers, $m$ and $n$ are relatively prime, and $p$ is not divisible by the square of any prime.


Find $m+n+p$.

Answer:

