2019 AIME II

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Question 1

Not yet answered

Points out of 1

Two different points, C and D, lie on the same side of line AB so that $\triangle ABC$ and $\triangle BAD$ are congruent with $AB = 9$, $BC = AD = 10$, and $CA = DB = 17$. The intersection of these two triangular regions has area $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 2

Not yet answered

Points out of 1

Lily pads $1, 2, 3, \ldots$ lie in a row on a pond. A frog makes a sequence of jumps starting on pad 1. From any pad k the frog jumps to either pad $k + 1$ or pad $k + 2$ chosen randomly with probability $\frac{1}{2}$ and independently of other jumps. The probability that the frog visits pad 7 is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Answer:

Question 3

Not yet answered

Points out of 1

Find the number of 7-tuples of positive integers (a, b, c, d, e, f, g) that satisfy the following systems of equations:

$$\begin{aligned} abc &= 70, \\ cde &= 71, \\ efg &= 72. \end{aligned}$$

Answer:

Question 4

Not yet answered

Points out of 1

A standard six-sided fair die is rolled four times. The probability that the product of all four numbers rolled is a perfect square is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer:

Question 5

Not yet answered

Points out of 1

Four ambassadors and one advisor for each of them are to be seated at a round table with 12 chairs numbered in order 1 to 12. Each ambassador must sit in an even-numbered chair. Each advisor must sit in a chair adjacent to his or her ambassador. There are N ways for the 8 people to be seated at the table under these conditions. Find the remainder when N is divided by 1000.

Answer:

Question 6

Not yet answered

Points out of 1

In a Martian civilization, all logarithms whose bases are not specified as assumed to be base b, for some fixed $b \geq 2$. A Martian student writes down

$$\begin{aligned} 3\log(\sqrt{x}\log x) &= 56 \\ \log_{\log x}(x) &= 54 \end{aligned}$$

and finds that this system of equations has a single real number solution $x > 1$. Find b.

Answer:

Question 7

Not yet answered

Points out of 1

Triangle ABC has side lengths $AB = 120$, $BC = 220$, and $AC = 180$. Lines ℓ_A ,ℓ_B, and ℓ_C are drawn parallel to $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively, such that the intersections of ℓ_A, ℓ_B, and ℓ_C with the interior of $\triangle ABC$ are segments of lengths 55, 45, and 15, respectively. Find the perimeter of the triangle whose sides lie on lines ℓ_A, ℓ_B, and ℓ_C.

Answer:

Question 8

Not yet answered

Points out of 1

The polynomial $f(z) = az^{2018} + bz^{2017} + cz^{2016}$ has real coefficients not exceeding 2019, and $f\left(\frac{1+\sqrt{3}i}{2}\right) = 2015 + 2019\sqrt{3}i$. Find the remainder when $f(1)$ is divided by 1000.

Answer:

Question 9

Not yet answered

Points out of 1

Call a positive integer n *k-pretty* if n has exactly k positive divisors and n is divisible by k. For example, 18 is 6-pretty. Let S be the sum of positive integers less than 2019 that are 20-pretty. Find $\frac{S}{20}$.

Answer:

Question **10**

Not yet answered

Points out of 1

There is a unique angle θ between 0° and 90° such that for nonnegative integers n, the value of $\tan(2^n\theta)$ is positive when n is a multiple of 3, and negative otherwise. The degree measure of θ is $\frac{p}{q}$, where p and q are relatively prime integers. Find $p+q$.

Answer:

Question **11**

Not yet answered

Points out of 1

Triangle ABC has side lengths $AB=7$, $BC=8$, and $CA=9$. Circle ω_1 passes through B and is tangent to line AC at A. Circle ω_2 passes through C and is tangent to line AB at A. Let K be the intersection of circles ω_1 and ω_2 not equal to A. Then $AK=\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Answer:

Question **12**

Not yet answered

Points out of 1

For $n\geq 1$ call a finite sequence $(a_1, a_2\ldots, a_n)$ of positive integers *progressive* if $a_i < a_{i+1}$ and a_i divides a_{i+1} for all $1\leq i\leq n-1$. Find the number of progressive sequences such that the sum of the terms in the sequence is equal to 360.

Answer:

Question **13**

Not yet answered

Points out of 1

Regular octagon $A_1A_2A_3A_4A_5A_6A_7A_8$ is inscribed in a circle of area 1. Point P lies inside the circle so that the region bounded by $\overline{PA_1}$, $\overline{PA_2}$, and the minor arc $\widehat{A_1A_2}$ of the circle has area $\frac{1}{7}$, while the region bounded by $\overline{PA_3}$, $\overline{PA_4}$, and the minor arc $\widehat{A_3A_4}$ of the circle has area $\frac{1}{9}$. There is a positive integer n such that the area of the region bounded by $\overline{PA_6}$, $\overline{PA_7}$, and the minor arc $\widehat{A_6A_7}$ of the circle is equal to $\frac{1}{8}-\frac{\sqrt{2}}{n}$. Find n.

Answer:

Question **14**

Not yet answered

Points out of 1

Find the sum of all positive integers n such that, given an unlimited supply of stamps of denominations 5, n, and $n+1$ cents, 91 cents is the greatest postage that cannot be formed.

Answer:

Question 15

Not yet answered

Points out of 1

In acute triangle $\triangle ABC$ points P and Q are the feet of the perpendiculars from C to $\overline{AB}$ and from B to $\overline{AC}$, respectively. Line PQ intersects the circumcircle of $\triangle ABC$ in two distinct points, X and Y. Suppose $XP = 10$, $PQ = 25$, and $QY = 15$. The value of $AB \cdot AC$ can be written in the form $m\sqrt{n}$ where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.

Answer: