



2005 AMC 10A

For more practice and resources, visit ziml.aretteam.org

The problems in the AMC-Series Contests are copyrighted by American Mathematics Competitions at Mathematical Association of America (www.maa.org).



Question 1

Not yet answered

Points out of 6

While eating out, Mike and Joe each tipped their server 2 dollars. Mike tipped 10% of his bill and Joe tipped 20% of his bill. What was the difference, in dollars between their bills?

(A) 2 (B) 4 (C) 5 (D) 10 (E) 20

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 2

Not yet answered

Points out of 6

For each pair of real numbers $a \neq b$, define the operation \star as

$$(a \star b) = \frac{a + b}{a - b}.$$

What is the value of $((1 \star 2) \star 3)$?

(A) $-\frac{2}{3}$ (B) $-\frac{1}{5}$ (C) 0 (D) $\frac{1}{2}$ (E) This value is not defined.

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 3

Not yet answered

Points out of 6

The equations $2x + 7 = 3$ and $bx - 10 = -2$ have the same solution. What is the value of b ?

- (A) -8 (B) -4 (C) 2 (D) 4 (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

A rectangle with a diagonal of length x is twice as long as it is wide. What is the area of the rectangle?

- (A) $\frac{1}{4}x^2$ (B) $\frac{2}{5}x^2$ (C) $\frac{1}{2}x^2$ (D) x^2 (E) $\frac{3}{2}x^2$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

A store normally sells windows at 100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How many dollars will they save if they purchase the windows together rather than separately?

- (A) 100 (B) 200 (C) 300 (D) 400 (E) 500

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

(A) 23 (B) 24 (C) 25 (D) 26 (E) 27

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

Josh and Mike live 13 miles apart. Yesterday Josh started to ride his bicycle toward Mike's house. A little later Mike started to ride his bicycle toward Josh's house. When they met, Josh had ridden for twice the length of time as Mike and at four-fifths of Mike's rate. How many miles had Mike ridden when they met?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Select one:

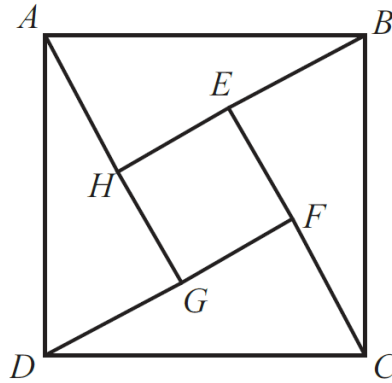
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

In the figure, the length of side AB of square $ABCD$ is $\sqrt{50}$ and $BE=1$.



What is the area of the inner square $EFGH$?

- (A) 25 (B) 32 (C) 36 (D) 40 (E) 42

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

Three tiles are marked X and two other tiles are marked O . The five tiles are randomly arranged in a row. What is the probability that the arrangement reads $XOXOX$?

- (A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x . What is the sum of those values of a ?

- (A) -16 (B) -8 (C) 0 (D) 8 (E) 20

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n ?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Select one:

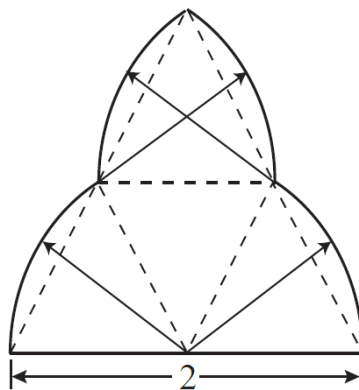
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

The figure shown is called a *trefoil* and is constructed by drawing circular sectors about the sides of the congruent equilateral triangles.



What is the area of a trefoil whose horizontal base has length 2?

- (A) $\frac{1}{3}\pi + \frac{\sqrt{3}}{2}$ (B) $\frac{2}{3}\pi$ (C) $\frac{2}{3}\pi + \frac{\sqrt{3}}{4}$ (D) $\frac{2}{3}\pi + \frac{\sqrt{3}}{3}$ (E) $\frac{2}{3}\pi + \frac{\sqrt{3}}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

How many positive integers n satisfy the following condition:

$$(130n)^{50} > n^{100} > 2^{200}?$$

- (A) 0 (B) 7 (C) 12 (D) 65 (E) 125

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

- (A) 41 (B) 42 (C) 43 (D) 44 (E) 45

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 15

Not yet answered

Points out of 6

How many positive cubes divide $3! \cdot 5! \cdot 7!$?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

The sum of the digits of a two-digit number is subtracted from the number. The units digit of the result is 6. How many two-digit numbers have this property?

- (A) 5 (B) 7 (C) 9 (D) 10 (E) 19

Select one:

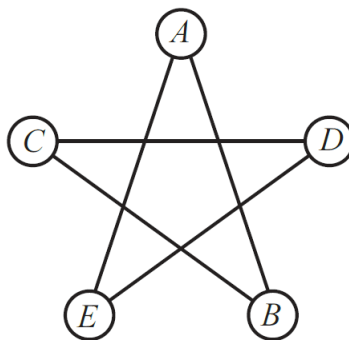
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 17

Not yet answered

Points out of 6

In the five-sided star shown, the letters A , B , C , D , and E are replaced by the numbers 3, 5, 6, 7, and 9, although not necessarily in this order. The sums of the numbers at the ends of the line segments AB , BC , CD , DE , and EA form an arithmetic sequence, although not necessarily in that order.



What is the middle term of the arithmetic sequence?

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

Team A and team B play a series. The first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If team B wins the second game and team A wins the series, what is the probability that team B wins the first game?

- (A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Select one:

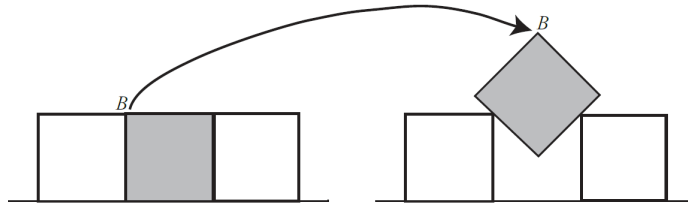
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

Three one-inch squares are placed with their bases on a line. The center square is lifted out and rotated 45 degrees, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares.



How many inches is the point B from the line on which the bases of the original squares were placed?

- (A) 1 (B) $\sqrt{2}$ (C) $\frac{3}{2}$ (D) $\sqrt{2} + \frac{1}{2}$ (E) 2

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

An equiangular octagon has four sides of length 1 and four sides of length $\frac{\sqrt{2}}{2}$, arranged so that no two consecutive sides have the same length. What is the area of the octagon?

- (A) $\frac{7}{2}$ (B) $\frac{7\sqrt{2}}{2}$ (C) $\frac{5 + 4\sqrt{2}}{2}$ (D) $\frac{4 + 5\sqrt{2}}{2}$ (E) 7

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

For how many positive integers n does $1 + 2 + \dots + n$ evenly divide from $6n$?

(A) 3 (B) 5 (C) 7 (D) 9 (E) 11

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 22

Not yet answered

Points out of 6

Let S be the set of the 2005 smallest positive multiples of 4, and let T be the set of the 2005 smallest positive multiples of 6. How many elements are common to S and T ?

(A) 166 (B) 333 (C) 500 (D) 668 (E) 1001

Select one:

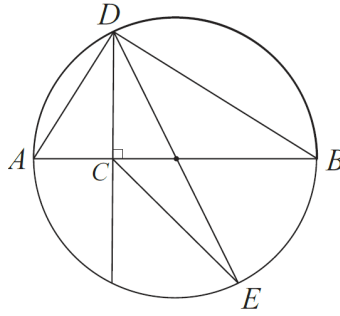
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

Let AB be a diameter of a circle and let C be a point on AB with $2 \cdot AC = BC$. Let D and E be points on the circle such that $DC \perp AB$ and DE is a second diameter.



What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?

- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Select one:

- A
 B
 C
 D
 E
 Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

For each positive integer $n > 1$, let $P(n)$ denote the greatest prime factor of n . For how many positive integers n is it true that both $P(n) = \sqrt{n}$ and $P(n + 48) = \sqrt{n + 48}$?

- (A) 0 (B) 1 (C) 3 (D) 4 (E) 5

Select one:

- A
 B
 C
 D
 E
 Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

In ABC we have $AB = 25$, $BC = 39$, and $AC = 42$. Points D and E are on AB and AC respectively, with $AD = 19$ and $AE = 14$. What is the ratio of the area of triangle ADE to the area of the quadrilateral $BCED$?

- (A) $\frac{266}{1521}$ (B) $\frac{19}{75}$ (C) $\frac{1}{3}$ (D) $\frac{19}{56}$ (E) 1

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)