



# 2018 AMC 10B

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**Question 1**

Not yet answered

Points out of 6

Kate bakes 20-inch by 18-inch pan of cornbread. The cornbread is cut into pieces that measure 2 inches by 2 inches. How many pieces of cornbread does the pan contain?

(A) 90    (B) 100    (C) 180    (D) 200    (E) 360

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 2**

Not yet answered

Points out of 6

Sam drove 96 miles in 90 minutes. His average speed during the first 30 minutes was 60 mph (miles per hour), and his average speed during the second 30 minutes was 65 mph. What was his average speed, in mph, during the last 30 minutes?

(A) 64    (B) 65    (C) 66    (D) 67    (E) 68

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 3**

Not yet answered

Points out of 6

In the expression  $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$  each blank is to be filled in with one of the digits 1, 2, 3, or 4, with each digit being used once. How many different values can be obtained?

(A) 2      (B) 3      (C) 4      (D) 6      (E) 24

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 4**

Not yet answered

Points out of 6

A three-dimensional rectangular box with dimensions  $X$ ,  $Y$ , and  $Z$  has faces whose surface areas are 24, 24, 48, 48, 72, and 72 square units. What is  $X + Y + Z$ ?

(A) 18      (B) 22      (C) 24      (D) 30      (E) 36

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 5**

Not yet answered

Points out of 6

How many subsets of  $\{2, 3, 4, 5, 6, 7, 8, 9\}$  contain at least one prime number?

(A) 128      (B) 192      (C) 224      (D) 240      (E) 256

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 6**

Not yet answered

Points out of 6

A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

- (A)  $\frac{1}{15}$     (B)  $\frac{1}{10}$     (C)  $\frac{1}{6}$     (D)  $\frac{1}{5}$     (E)  $\frac{1}{4}$

Select one:

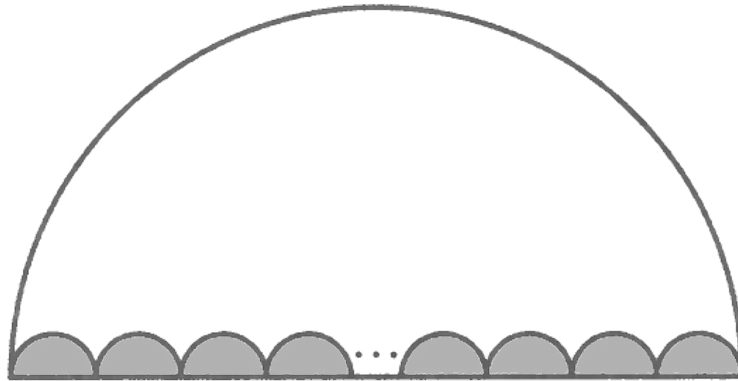
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 7**

Not yet answered

Points out of 6

In the figure below,  $N$  congruent semicircles are drawn along a diameter of a large semicircle, with their diameters covering the diameter of the large semicircle with no overlap. Let  $A$  be the combined area of the small semicircles and  $B$  be the area of the region inside the large semicircle but outside the semicircles. The ratio  $A : B$  is 1 : 18. What is  $N$ ?



- (A) 16    (B) 17    (C) 18    (D) 19    (E) 36

Select one:

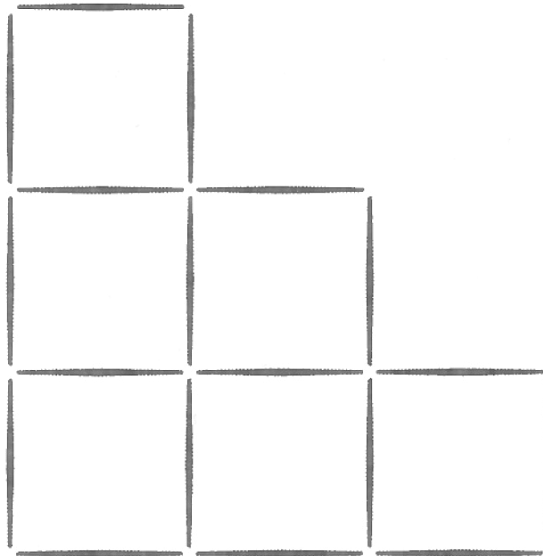
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 8**

Not yet answered

Points out of 6

Sara makes a staircase out of toothpicks as shown



This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

(A) 10    (B) 11    (C) 12    (D) 24    (E) 30

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 9**

Not yet answered

Points out of 6

The faces of each of 7 standard dice are labeled with the integers from 1 to 6. Let  $p$  be the probability that when all 7 dice are rolled, the sum of the numbers on the top faces is 10. What other sum occurs with the same probability  $p$ ?

(A) 13    (B) 26    (C) 32    (D) 39    (E) 42

Select one:

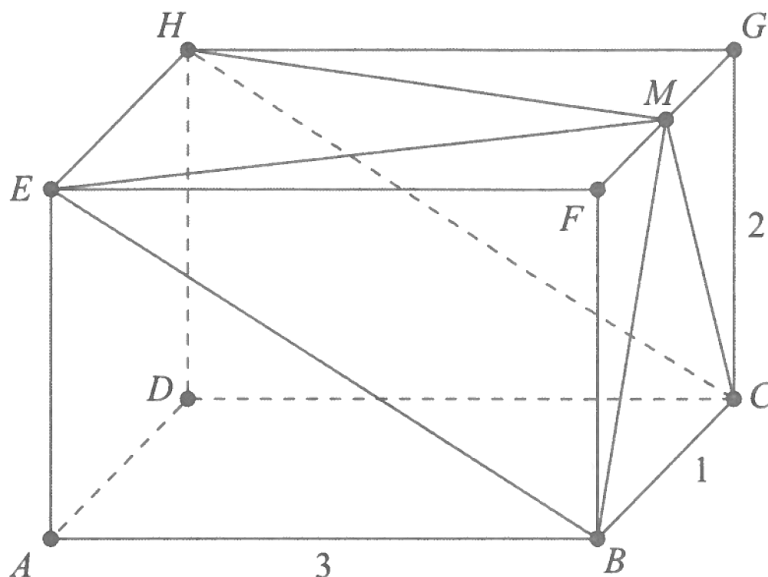
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 10**

Not yet answered

Points out of 6

In the rectangular parallelepiped shown,  $AB = 3$ ,  $BC = 1$ , and  $CG = 2$ . Point  $M$  is the midpoint of  $\overline{FG}$ . What is the volume of the rectangular pyramid with base  $BCHE$  and apex  $M$ ?



- (A) 1      (B)  $\frac{4}{3}$       (C)  $\frac{3}{2}$       (D)  $\frac{5}{3}$       (E) 2

Select one:

- A  
 B  
 C  
 D  
 E  
 Leave blank (1.5 points)

**Question 11**

Not yet answered

Points out of 6

Which of the following expressions is never a prime number when  $p$  is a prime number?

- (A)  $p^2 + 16$       (B)  $p^2 + 24$       (C)  $p^2 + 26$       (D)  $p^2 + 46$       (E)  $p^2 + 96$

Select one:

- A  
 B  
 C  
 D  
 E  
 Leave blank (1.5 points)

**Question 12**

Not yet answered

Points out of 6

Line segment  $\overline{AB}$  is a diameter of a circle with  $AB = 24$ . Point  $C$ , not equal to  $A$  or  $B$ , lies on the circle. As point  $C$  moves around the circle, the centroid (center of mass) of  $\triangle ABC$  traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve?

- (A) 25      (B) 38      (C) 50      (D) 63      (E) 75

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 13**

Not yet answered

Points out of 6

How many of the first 2018 numbers in the sequence 101, 1001, 10001, 100001, ... are divisible by 101?

- (A) 253      (B) 504      (C) 505      (D) 506      (E) 1009

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 14**

Not yet answered

Points out of 6

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

- (A) 202      (B) 223      (C) 224      (D) 225      (E) 234

Select one:

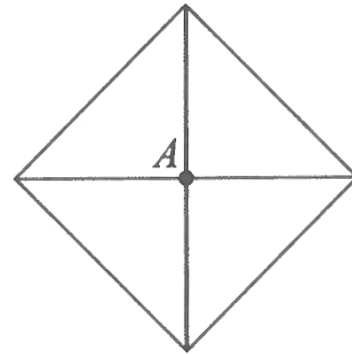
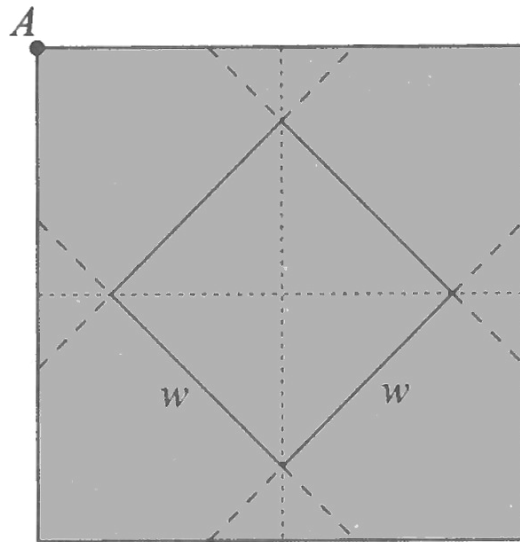
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 15**

Not yet answered

Points out of 6

A closed box with a square base is to be wrapped with a square sheet of wrapping paper. The box is centered on the wrapping paper with the vertices of the base lying on the midlines of the square sheet of paper, as shown in the figure on the left. The four corners of the wrapping paper are to be folded up over the sides and brought together to meet at the center of the top of the box, point  $A$  in the figure on the right. The box has base length  $w$  and height  $h$ . What is the area of the sheet of wrapping paper?



- (A)  $2(w + h)^2$     (B)  $\frac{(w+h)^2}{2}$     (C)  $2w^2 + 4wh$     (D)  $2w^2$     (E)  $w^2h$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)



**Question 16**

Not yet answered

Points out of 6

Let  $a_1, a_2, \dots, a_{2018}$  be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \dots + a_{2018} = 2018^{2018}.$$

What is the remainder when  $a_1^3 + a_2^3 + \dots + a_{2018}^3$  is divided by 6?

- (A) 0    (B) 1    (C) 2    (D) 3    (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 17**

Not yet answered

Points out of 6

In rectangle  $PQRS$ ,  $PQ = 8$  and  $QR = 6$ . Points  $A$  and  $B$  lie on  $\overline{PQ}$ , points  $C$  and  $D$  lie on  $\overline{QR}$ , points  $E$  and  $F$  lie on  $\overline{RS}$ , and points  $G$  and  $H$  lie on  $\overline{SP}$  so that  $AP = BQ < 4$  and the convex octagon  $ABCDEFGH$  is equilateral. The length of a side of this octagon can be expressed in the form  $k + m\sqrt{n}$ , where  $k$ ,  $m$ , and  $n$  are integers and  $n$  is not divisible by the square of any prime. What is  $k + m + n$ ?

- (A) 1    (B) 7    (C) 21    (D) 92    (E) 106

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 18**

Not yet answered

Points out of 6

Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, siblings may not sit right next to each other in the same row, and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

(A) 60    (B) 72    (C) 92    (D) 96    (E) 120

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 19**

Not yet answered

Points out of 6

Joey and Chloe and their daughter Zoe all have the same birthday. Joey is 1 year older than Chloe, and Zoe is exactly 1 year old today. Today is the first of the 9 birthdays on which Chloe's age will be an integral multiple of Zoe's age. What will be the sum of the two digits of Joey's age the next time his age is a multiple of Zoe's age?

(A) 7    (B) 8    (C) 9    (D) 10    (E) 11

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 20**

Not yet answered

Points out of 6

A function  $f$  is defined recursively by  $f(1) = f(2) = 1$  and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers  $n \geq 3$ . What is  $f(2018)$ ?

- (A) 2016      (B) 2017      (C) 2018      (D) 2019      (E) 2020

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 21**

Not yet answered

Points out of 6

Mary chose an even 4-digit number  $n$ . She wrote down all the divisors of  $n$  in increasing order from left to right:  $1, 2, \dots, \frac{n}{2}, n$ . At some moment Mary wrote 323 as a divisor of  $n$ .

What is the smallest possible value of the next divisor written to the right of 323?

- (A) 324      (B) 330      (C) 340      (D) 361      (E) 646

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 22**

Not yet answered

Points out of 6

Real numbers  $x$  and  $y$  are chosen independently and uniformly at random from the interval  $[0, 1]$ . Which of the following numbers is closest to the probability that  $x$ ,  $y$ , and 1 are the side lengths of an obtuse triangle?

(A) 0.21      (B) 0.25      (C) 0.29      (D) 0.50      (E) 0.79

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 23**

Not yet answered

Points out of 6

How many ordered pairs  $(a, b)$  of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \text{gcd}(a, b),$$

where  $\text{gcd}(a, b)$  denotes the greatest common divisor of  $a$  and  $b$ , and  $\text{lcm}(a, b)$  denotes their least common multiple?

(A) 0      (B) 2      (C) 4      (D) 6      (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 24**

Not yet answered

Points out of 6

Let  $ABCDEF$  be a regular hexagon with side length 1. Denote by  $X$ ,  $Y$ , and  $Z$  the midpoints of sides  $\overline{AB}$ ,  $\overline{CD}$ , and  $\overline{EF}$ , respectively. What is the area of the convex hexagon whose interior is the intersection of the interiors of  $\triangle ACE$  and  $\triangle XYZ$ ?

- (A)  $\frac{3}{8}\sqrt{3}$     (B)  $\frac{7}{16}\sqrt{3}$     (C)  $\frac{15}{32}\sqrt{3}$     (D)  $\frac{1}{2}\sqrt{3}$     (E)  $\frac{9}{16}\sqrt{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 25**

Not yet answered

Points out of 6

Let  $\lfloor x \rfloor$  denote the greatest integer less than or equal to  $x$ . How many real numbers  $x$  satisfy the equation  $x^2 + 10,000\lfloor x \rfloor = 10,000x$ ?

- (A) 197    (B) 198    (C) 199    (D) 200    (E) 201

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)