

# 2024 AMC 10B

Try this exam as a timed Mock Exam on the <a href="ZIML Practice Page">ZIML Practice Page (click here)</a>

View answers and concepts tested in our 2024 AMC 10B+12B Blog Post (click here)

The problems in the AMC-Series Contests are copyrighted by American Mathematics Competitions at Mathematical Association of America (www.maa.org).



Question 1  Not yet answered		n a long line of people arranged left to right, the 1013th person from the left is also the 1010th person from the right. How many people are in the line?						
Points out of 6	(A) 2021	<b>(B)</b> 2022	(C) 2023	<b>(D)</b> 2024	<b>(E)</b> 2025			
	Select one:      A     B     C     D     E	nk (1.5 points						
Question 2  Not yet answered	What is $10!$ $-$	7! · 6!? ( <b>B</b> ) 0	(C) 120	( <b>D</b> ) 600 ( <b>E</b>	S) 720			
Points out of 6		(2) 0	(3) 120	(2) 000 (2	., 120			
	Select one:							
	○ B							
	○ <b>C</b>							
	○ <b>D</b>							
	○ <b>E</b>							
	○ Leave bla	nk (1.5 points	)					
Question 3	For how many	integer values	of $x$ is $ 2x  \leq 1$	7π?				
Not yet answered				20 <b>(E)</b> 21				
Points out of 6	Select one:							
	○ A							
	○ <b>B</b>							
	○ <b>C</b>							
	$\bigcirc$ D							
	○ <b>E</b>							
	○ Leave bla	nk (1.5 points	)					

Not yet answered

Points out of 6

Balls numbered  $1,2,3,\ldots$  are deposited in 5 bins, labeled A, B, C, D, and E, using the following procedure. Ball 1 is deposited in bin A, and balls 2 and 3 are deposited in bin B. The next 3 balls are deposited in bin C, the next 4 in bin D, and so on, cycling back to bin A after balls are deposited in bin E. (For example, balls numbered  $22,23,\ldots,28$  are deposited in bin B at step 7 of this process.) In which bin is ball 2024 deposited?

- **(A)** A
- **(B)** *B*
- $(\mathbf{C})$  C
- **(D)** *D*
- $(\mathbf{E}) E$

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

## Question 5

Not yet answered

Points out of 6

In the following expression, Melanie changed some of the plus signs to minus signs:

$$1+3+5+7+\ldots+97+99$$

When the new expression was evaluated, it was negative. What is the least number of plus signs that Melanie could have changed to minus signs?

- **(A)** 14
- **(B)** 15
- **(C)** 16
- **(D)** 17
- **(E)** 18

- A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\cap$  E
- Leave blank (1.5 points)

Question 6	A rectangle has integer length sides and an area of $2024$ . What is the least possible perimeter of the rectangle?
Not yet answered	(A) 160 (B) 180 (C) 222 (D) 228 (E) 390
Points out of 6	$(\mathbf{A}) 100  (\mathbf{D}) 100  (\mathbf{C}) 222  (\mathbf{D}) 220  (\mathbf{E}) 390$
	Select one:
	$\bigcirc$ A
	○ <b>B</b>
	○ <b>C</b>
	○ <b>D</b>
	○ <b>E</b>
	○ Leave blank (1.5 points)
Question 7	What is the remainder when $7^{2024}+7^{2025}+7^{2026}$ is divided by $19$ ?
Not yet answered	(A) $0$ (B) $1$ (C) $7$ (D) $11$ (E) $18$
Points out of 6	Select one:
	○ <b>A</b>
	○ <b>B</b>
	○ <b>C</b>
	○ <b>D</b>
	○ <b>E</b>
	○ Leave blank (1.5 points)
Question 8	Let $N$ be the product of all the positive integer divisors of $42$ . What is the units digit of $N$ ?
Not yet answered	<b>(A)</b> $0$ <b>(B)</b> $2$ <b>(C)</b> $4$ <b>(D)</b> $6$ <b>(E)</b> $8$
Points out of 6	Select one:
	○ <b>A</b>
	○ <b>B</b>
	○ <b>C</b>
	$\bigcirc$ D
	○ <b>E</b>
	○ Leave blank (1.5 points)

Not yet answered

Points out of 6

Real numbers  $a,\,b,\,$  and c have arithmetic mean 0. The arithmetic mean of  $a^2,\,b^2,\,$  and  $c^2$  is 10. What is the arithmetic mean of ab, ac, and bc?

$$(A) - 5$$

**(B)** 
$$-\frac{10}{3}$$

(A) 
$$-5$$
 (B)  $-\frac{10}{3}$  (C)  $-\frac{10}{9}$  (D) 0 (E)  $\frac{10}{9}$ 

**(E)** 
$$\frac{10}{9}$$

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

## Question 10

Not yet answered

Points out of 6

Quadrilateral ABCD is a parallelogram, and E is the midpoint of the side AD. Let F be the intersection of lines EB and AC. What is the ratio of the area of quadrilateral CDEFto the area of  $\triangle CFB$ ?

**(A)** 
$$5:4$$

**(B)** 
$$4:3$$

**(B)** 
$$4:3$$
 **(C)**  $3:2$  **(D)**  $5:3$  **(E)**  $2:1$ 

**(D)** 
$$5:3$$

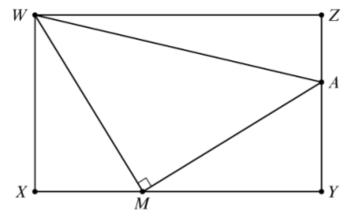
**(E)** 
$$2:1$$

- $\bigcirc$  A
- $\bigcirc$  B
- $\circ$  c
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

Not yet answered

Points out of 6

In the figure below WXYZ is a rectangle with WX=4 and WZ=8. Point M lies  $\overline{XY}$ , point A lies on  $\overline{YZ}$ , and  $\angle WMA$  is a right angle. The areas of  $\triangle WXM$  and  $\triangle WAZ$  are equal. What is the area of  $\triangle WMA$ ?



- **(A)** 13
- **(B)** 14
- **(C)** 15
- **(D)** 16
- **(E)** 17

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- E
- Leave blank (1.5 points)

#### Question 12

Not yet answered

Points out of 6

A group of 100 students from different countries meet at a mathematics competition. Each student speaks the same number of languages, and, for every pair of students A and B, student A speaks some language that student B does not speak, and student B speaks some language that student A does not speak. What is the least possible total number of languages spoken by all the students?

- **(A)** 9
- **(B)** 10
- **(C)** 12
- **(D)** 51
- **(E)** 100

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\cap$  E
- Leave blank (1.5 points)

Not yet answered

Points out of 6

Positive integers x and y satisfy the equation  $\sqrt{x} + \sqrt{y} = \sqrt{1183}$ . What is the minimum possible value of x + y?

**(A)** 585

**(B)** 595

**(C)** 623

**(D)** 700

**(E)** 791

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

## Question 14

Not yet answered

Points out of 6

A dartboard is the region B in the coordinate plane consisting of points (x,y) such that  $|x|+|y|\leq 8$ . A target T is the region where  $(x^2+y^2-25)^2\leq 49$ . A dart is thrown and lands at a random point in B. The probability that the dart lands in T can be expressed as  $rac{m}{n}\cdot\pi$ , where m and n are relatively prime positive integers. What is m+n?

(A) 39

**(B)** 71

**(C)** 73

**(D)** 75

(E) 135

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

## Question 15

Not yet answered

Points out of 6

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y, and z with  $x \leq y \leq z$ . The range of the list is 7, and the mean and the median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1

**(B)** 2

**(C)** 3 **(D)** 4 **(E)** infinitely many

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\cap$  E
- Leave blank (1.5 points)

Question	1	6
----------	---	---

Not yet answered

Points out of 6

Jerry likes to play with numbers. One day, he wrote all the integers from 1 to 2024 on the whiteboard. Then he repeatedly chose four numbers on the whiteboard, erased them, and replaced them by either their sum or their product. (For example, Jerry's first step might have been to erase  $1,\,2,\,3,\,$  and  $5,\,$  and then write either  $11,\,$  their sum, or  $30,\,$  their product, on the whiteboard.) After repeatedly performing this operation, Jerry noticed that all the remaining numbers on the whiteboard were odd. What is the maximum possible number of integers on the whiteboard at that time?

**(A)** 1010

**(B)** 1011

**(C)** 1012

**(D)** 1013

**(E)** 1014

Select one:

○ A

○ B

 $\bigcirc$  C

 $\bigcirc$  D

○ E

○ Leave blank (1.5 points)

#### Question 17

Not yet answered

Points out of 6

In a race among 5 snails, there is at most one tie, but that tie can involve any number of snails. For example, the result might be that Dazzler is first; Abby, Cyrus, and Elroy are tied for second; and Bruna is fifth. How many different results of the race are possible?

**(A)** 180

**(B)** 361

(C) 420

**(D)** 431

(E) 720

Select one:

 $\bigcirc$  A

 $\bigcirc$  B

 $\bigcirc$  C

 $\bigcirc$  D

 $\bigcirc$  E

Leave blank (1.5 points)

Question 18	How man 125?	y different remain	ders can re	esult when th	100th pow	er of an integer i	is divided by
Not yet answered							
Points out of 6	( <b>A</b> ) 1	(B) 2 (C	(I	O) 25 (	<b>E</b> ) 125		
	Select on	e:					
	○ A						
	ОВ						
	○ <b>c</b>						
	○ <b>D</b>						
	○ <b>E</b>						
	○ Leav	e blank (1.5 poin	its)				
Question 19	In the follo	owing table, each	question m	nark is to be	replaced by "	Possible" or "No	ot Possible"
Not yet answered		e whether a nonve					
Points out of 6	lattice poi will be "Po	ints (points both o	f whose co	ordinates are	e integers). H	ow many of the	12 entries
	will be Po	ossible ?					_
			zero	exactly one	exactly two	more than two	
				one	two	two	
		zero slope	?	?	?	?	
			+				1
		nonzero rational slope	?	?	?	?	
		rational	?	?	?	?	
	(4) 4	rational slope irrational slope	?	•	•	?	
	(A) 4	rational slope irrational	?	•	? ? E) 9	?	
	Select on	rational slope  irrational slope  (B) 5 (C	?	•	•	?	
		rational slope  irrational slope  (B) 5 (C	?	•	•	?	
	Select on	rational slope  irrational slope  (B) 5 (C	?	•	•	?	
	Select on	rational slope  irrational slope  (B) 5 (C	?	•	•	?	
	Select on A	rational slope  irrational slope  (B) 5 (C	?	•	•	?	

○ Leave blank (1.5 points)

Not yet answered

Points out of 6

from a different pair. In how many ways can these six shoes be lined up?

**(D)** 108 **(E)** 120

Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe

Select one:

 $\bigcirc$  A

**(A)** 60

- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

**(B)** 72

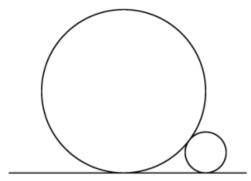
**(C)** 90

## Question 21

Not yet answered

Points out of 6

Two straight pipes (circular cylinders), with radii 1 and  $\frac{1}{4}$ , lie parallel and in contact on a flat floor. The figure below shows a head-on view. What is the sum of the possible radii of a third parallel pipe lying on the same floor and in contact with both?



- (A)  $\frac{1}{9}$  (B) 1 (C)  $\frac{10}{9}$  (D)  $\frac{11}{9}$  (E)  $\frac{19}{9}$

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\cap$  E
- Leave blank (1.5 points)

Not yet answered

Points out of 6

A group of 16 people will be partitioned into 4 indistinguishable 4-person committees. Each committee will have one chairperson and one secretary. The number of different ways to make these assignments can be written as  $3^rM$ , where r and M are positive integers and M is not divisible by 3. What is r?

- (A) 5
- **(B)** 6
- $(C) 7 \qquad (D) 8$
- **(E)** 9

Select one:

- $\bigcirc$  A
- $\bigcirc$  B
- $\circ$  c
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

## Question 23

Not yet answered

Points out of 6

The Fibonacci numbers are defined by  $F_1=1$ ,  $F_2=1$ , and  $F_n=F_{n-1}+F_{n-2}$  for  $n\geq 3$ . What is

$$\frac{F_2}{F_1} + \frac{F_4}{F_2} + \frac{F_6}{F_3} + \dots + \frac{F_{20}}{F_{10}}$$
?

- (A) 318
- **(B)** 319
- (C) 320 (D) 321
- **(E)** 322

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)

Not yet answered

Points out of 6

Let

$$P(m) = rac{m}{2} + rac{m^2}{4} + rac{m^4}{8} + rac{m^8}{8}.$$

How many of the values  $P(2022),\,P(2023),\,P(2024),\,{\rm and}\,\,P(2025)$  are integers?

(A) 0

**(B)** 1

(C) 2 (D) 3

 $(\mathbf{E}) \ 4$ 

Select one:

 $\bigcirc$  A

 $\bigcirc$  B

 $\bigcirc$  C

 $\bigcirc$  D

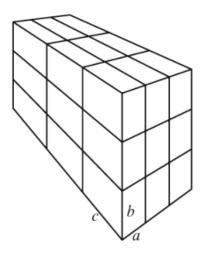
 $\bigcirc$  E

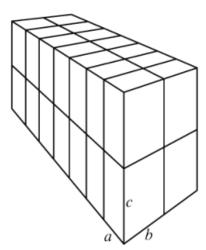
○ Leave blank (1.5 points)

Not yet answered

Points out of 6

Each of 27 bricks (right rectangular prisms) has dimensions  $a \times b \times c$ , where a,b, and c are pairwise relatively prime positive integers. These bricks are arranged to form a  $3 \times 3 \times 3$  block, as shown on the left below. A 28th brick with the same dimensions is introduced, and these bricks are reconfigured into a  $2 \times 2 \times 7$  block, shown on the right. The new block is 1 unit taller, 1 unit wider, and 1 unit deeper than the old one. What is a+b+c?





- **(A)** 88
- **(B)** 89
- **(C)** 90
- **(D)** 91
- **(E)** 92

- $\bigcirc$  A
- $\bigcirc$  B
- $\bigcirc$  C
- $\bigcirc$  D
- $\bigcirc$  E
- Leave blank (1.5 points)