2002 AMC 12B

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Question 1

Not yet answered

Points out of 6

The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. The number M does not contain the digit

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ 2 \qquad (\mathrm{C})\ 4 \qquad (\mathrm{D})\ 6 \qquad (\mathrm{E})\ 8$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

What is the value of $(3x-2)(4x+1)-(3x-2)4x+1$ when $x=4$?

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ 1 \qquad (\mathrm{C})\ 10 \qquad (\mathrm{D})\ 11 \qquad (\mathrm{E})\ 12$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

For how many positive integers n is $n^2 - 3n + 2$ a prime number?

(A) none
(B) one
(C) two
(D) more than two, but finitely many
(E) infinitely many

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

Let n be a positive integer such that $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is an integer. Which of the following statements is **not** true:

(A) 2 divides n
(B) 3 divides n
(C) 6 divides n
(D) 7 divides n
(E) $n > 84$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

Let $v, w, x, y,$ and z be the degree measures of the five angles of a pentagon. Suppose that $v < w < x < y < z$ and $v, w, x, y,$ and z form an arithmetic sequence. Find the value of x.

$(\mathrm{A})\ 72 \qquad (\mathrm{B})\ 84 \qquad (\mathrm{C})\ 90 \qquad (\mathrm{D})\ 108 \qquad (\mathrm{E})\ 120$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

Suppose that a and b are nonzero real numbers, and that the equation $x^2 + ax + b = 0$ has solutions a and b. Then the pair (a, b) is

$(\mathrm{A})\ (-2, 1) \qquad (\mathrm{B})\ (-1, 2) \qquad (\mathrm{C})\ (1, -2) \qquad (\mathrm{D})\ (2, -1) \qquad (\mathrm{E})\ (4, 4)$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?

$(\mathrm{A})\ 50 \qquad (\mathrm{B})\ 77 \qquad (\mathrm{C})\ 110 \qquad (\mathrm{D})\ 149 \qquad (\mathrm{E})\ 194$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

Suppose July of year N has five Mondays. Which of the following must occurs five times in the August of year N? (Note: Both months have 31 days.)

(A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

If a, b, c, d are positive real numbers such that a, b, c, d form an increasing arithmetic sequence and a, b, d form a geometric sequence, then $\frac{a}{d}$ is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 10

Not yet answered

Points out of 6

How many different integers can be expressed as the sum of three distinct members of the set $\{1, 4, 7, 10, 13, 16, 19\}$?

(A) 13 (B) 16 (C) 24 (D) 30 (E) 35

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 11

Not yet answered

Points out of 6

The positive integers A, B, $A-B$, and $A+B$ are all prime numbers. The sum of these four primes is

(A) even
(B) divisible by 3
(C) divisible by 5
(D) divisible by 7
(E) prime

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 12

Not yet answered

Points out of 6

For how many integers n is $\dfrac{n}{20-n}$ the square of an integer?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 10

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 13

Not yet answered

Points out of 6

The sum of 18 consecutive positive integers is a perfect square. The smallest possible value of this sum is

$(\mathrm{A})\ 169 \quad (\mathrm{B})\ 225 \quad (\mathrm{C})\ 289 \quad (\mathrm{D})\ 361 \quad (\mathrm{E})\ 441$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 14

Not yet answered

Points out of 6

Four distinct circles are drawn in a plane. What is the maximum number of points where at least two of the circles intersect?

$(\mathrm{A})\ 8 \quad (\mathrm{B})\ 9 \quad (\mathrm{C})\ 10 \quad (\mathrm{D})\ 12 \quad (\mathrm{E})\ 16$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 15

Not yet answered

Points out of 6

How many four-digit numbers N have the property that the three-digit number obtained by removing the leftmost digit is one ninth of N?

$(\mathrm{A})\ 4 \quad (\mathrm{B})\ 5 \quad (\mathrm{C})\ 6 \quad (\mathrm{D})\ 7 \quad (\mathrm{E})\ 8$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 16

Not yet answered

Points out of 6

Juan rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Amal rolls a fair six-sided die. What is the probability that the product of the two rolls is a multiple of 3?

$(\mathrm{A})\ \frac{1}{12}$ $(\mathrm{B})\ \frac{1}{3}$ $(\mathrm{C})\ \frac{1}{2}$ $(\mathrm{D})\ \frac{7}{12}$ $(\mathrm{E})\ \frac{2}{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 17

Not yet answered

Points out of 6

Andy's lawn has twice as much area as Beth's lawn and three times as much area as Carlos' lawn. Carlos' lawn mower cuts half as fast as Beth's mower and one third as fast as Andy's mower. If they all start to mow their lawns at the same time, who will finish first?

(A) Andy
(B) Beth
(C) Carlos
(D) Andy and Carlos tie for first.
(E) All three tie.

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 18

Not yet answered

Points out of 6

A point P is randomly selected from the rectangular region with vertices $(0,0), (2,0), (2,1), (0,1)$. What is the probability that P is closer to the origin than it is to the point $(3,1)$?

$(\mathrm{A})\ \frac{1}{2} \qquad (\mathrm{B})\ \frac{2}{3} \qquad (\mathrm{C})\ \frac{3}{4} \qquad (\mathrm{D})\ \frac{4}{5} \qquad (\mathrm{E})\ 1$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 19

Not yet answered

Points out of 6

If a, b, and c are positive real numbers such that $a(b+c) = 152, b(c+a) = 162$, and $c(a+b) = 170$, then abc is

$(\mathrm{A})\ 672 \qquad (\mathrm{B})\ 688 \qquad (\mathrm{C})\ 704 \qquad (\mathrm{D})\ 720 \qquad (\mathrm{E})\ 750$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 20

Not yet answered

Points out of 6

Let $\triangle XOY$ be a right-angled triangle with $m\angle XOY = 90^\circ$. Let M and N be the midpoints of legs OX and OY, respectively. Given that $XN = 19$ and $YM = 22$, find XY.

$(\mathrm{A})\ 24 \qquad (\mathrm{B})\ 26 \qquad (\mathrm{C})\ 28 \qquad (\mathrm{D})\ 30 \qquad (\mathrm{E})\ 32$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 21

Not yet answered

Points out of 6

For all positive integers n less than 2002, let

$$a_n = \begin{cases} 11, & \text{if} n \text{ is divisible by 13 and 14;} \\ 13, & \text{if } n \text{ is divisible by 14 and 11;} \\ 14, & \text{if } n \text{ is divisible by 11 and 13;} \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\sum_{n=1}^{2001} a_n$.

(A) 448 (B) 486 (C) 1560 (D) 2001 (E) 2002

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

For all integers n greater than 1, define $a_n = \dfrac{1}{\log_n 2002}$. Let $b = a_2 + a_3 + a_4 + a_5$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then $b - c$ equals

(A) -2 (B) -1 (C) $\dfrac{1}{2002}$ (D) $\dfrac{1}{1001}$ (E) $\dfrac{1}{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

In $\triangle ABC$, we have $AB = 1$ and $AC = 2$. Side $\overline{BC}$ and the median from A to $\overline{BC}$ have the same length. What is BC?

$(\mathrm{A})\ \frac{1+\sqrt{2}}{2} \qquad (\mathrm{B})\ \frac{1+\sqrt{3}}{2} \qquad (\mathrm{C})\ \sqrt{2} \qquad (\mathrm{D})\ \frac{3}{2} \qquad (\mathrm{E})\ \sqrt{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

A convex quadrilateral $ABCD$ with area 2002 contains a point P in its interior such that $PA = 24, PB = 32, PC = 28, PD = 45$. Find the perimeter of $ABCD$.

$\mathbf{(A)}\ 4\sqrt{2002}$
$\mathbf{(B)}\ 2\sqrt{8465}$
$\mathbf{(C)}\ 2(48+\sqrt{2002})$
$\mathbf{(D)}\ 2\sqrt{8633}$
$\mathbf{(E)}\ 4(36+\sqrt{113})$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 25

Not yet answered

Points out of 6

Let $f(x) = x^2 + 6x + 1$, and let R denote the set of points (x, y) in the coordinate plane such that

$$f(x) + f(y) \leq 0 \qquad \text{and} \qquad f(x) - f(y) \leq 0$$

The area of R is closest to

$(\mathrm{A})\ 21 \qquad (\mathrm{B})\ 22 \qquad (\mathrm{C})\ 23 \qquad (\mathrm{D})\ 24 \qquad (\mathrm{E})\ 25$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**