2004 AMC 12A

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Question 1

Not yet answered

Points out of 6

Alicia earns 20 dollars per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

$(\mathrm{A})\ 0.0029 \qquad (\mathrm{B})\ 0.029 \qquad (\mathrm{C})\ 0.29 \qquad (\mathrm{D})\ 2.9 \qquad (\mathrm{E})\ 29$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

On the AMC 12, each correct answer is worth 6 points, each incorrect answer is worth 0 points, and each problem left unanswered is worth 2.5 points. If Charlyn leaves 8 of the 25 problems unanswered, how many of the remaining problems must she answer correctly in order to score at least 100?

$(\mathrm{A})11 \qquad (\mathrm{B})13 \qquad (\mathrm{C})14 \qquad (\mathrm{D})16 \qquad (\mathrm{E})17$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

$(\mathrm{A})33 \qquad (\mathrm{B})49 \qquad (\mathrm{C})50 \qquad (\mathrm{D})99 \qquad (\mathrm{E})100$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters, and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-granddaughters. How many of Bertha's daughters and grand-daughters have no daughters?

$(\mathrm{A})\ 22 \qquad (\mathrm{B})\ 23 \qquad (\mathrm{C})\ 24 \qquad (\mathrm{D})\ 25 \qquad (\mathrm{E})\ 26$

Select one:

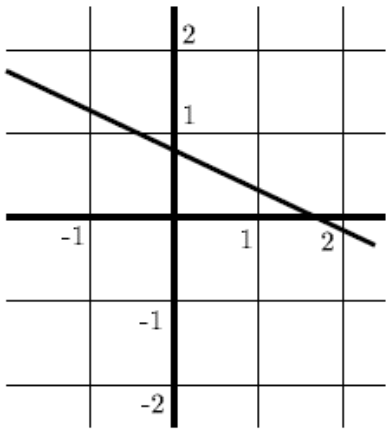
- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

The graph of the line $y = mx + b$ is shown.



Which of the following is true?

$(\mathrm{A}) mb < -1 \qquad (\mathrm{B})\ -1 < mb < 0 \qquad (\mathrm{C}) mb = 0 \qquad (\mathrm{D})\ 0 < mb < 1 \qquad (\mathrm{E}) mb > 1$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

Let $U = 2 \cdot 2004^{2005}$, $V = 2004^{2005}$, $W = 2003 \cdot 2004^{2004}$, $X = 2 \cdot 2004^{2004}$, $Y = 2004^{2004}$ and $Z = 2004^{2003}$. Which of the following is the largest?

$(\mathrm{A}) U - V \qquad (\mathrm{B}) V - W \qquad (\mathrm{C}) W - X \qquad (\mathrm{D}) X - Y \qquad (\mathrm{E}) Y - Z$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

A game is played with tokens according to the following rule. In each round, the player with the most tokens gives one token to each of the other players and also places one token in the discard pile. The game ends when some player runs out of tokens. Players A, B, and C start with 15, 14, and 13 tokens, respectively. How many rounds will there be in the game?

$(\mathrm{A})\ 36 \qquad (\mathrm{B})\ 37 \qquad (\mathrm{C})\ 38 \qquad (\mathrm{D})\ 39 \qquad (\mathrm{E})\ 40$

Select one:

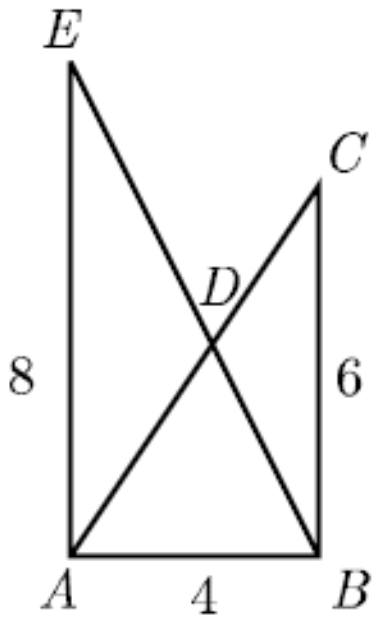
- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

In the overlapping triangles $\triangle ABC$ and $\triangle ABE$ sharing common side AB, $\angle EAB$ and $\angle ABC$ are right angles, $AB = 4$, $BC = 6$, $AE = 8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D.



What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?

$(\mathrm{A})\ 2 \qquad (\mathrm{B})\ 4 \qquad (\mathrm{C})\ 5 \qquad (\mathrm{D})\ 8 \qquad (\mathrm{E})\ 9$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

$(\mathrm{A})\ 10 \qquad (\mathrm{B})\ 25 \qquad (\mathrm{C})\ 36 \qquad (\mathrm{D})\ 50 \qquad (\mathrm{E})\ 60$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 10

Not yet answered

Points out of 6

The sum of 49 consecutive integers is 7^5. What is their median?

$(\mathrm{A})\ 7 \qquad (\mathrm{B})\ 7^2 \qquad (\mathrm{C})\ 7^3 \qquad (\mathrm{D})\ 7^4 \qquad (\mathrm{E})\ 7^5$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 11

Not yet answered

Points out of 6

The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

$(\mathrm{A})\ 0 \qquad (\mathrm{B})\ 1 \qquad (\mathrm{C})\ 2 \qquad (\mathrm{D})\ 3 \qquad (\mathrm{E})\ 4$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 12

Not yet answered

Points out of 6

Let $A = (0, 9)$ and $B = (0, 12)$. Points A' and B' are on the line $y = x$, and $\overline{AA'}$ and $\overline{BB'}$ intersect at $C = (2, 8)$. What is the length of $\overline{A'B'}$?

$(\mathrm{A})2 \qquad (\mathrm{B})2\sqrt{2} \qquad (\mathrm{C})3 \qquad (\mathrm{D})2+\sqrt{2} \qquad (\mathrm{E})3\sqrt{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 13

Not yet answered

Points out of 6

Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?

$(\mathrm{A})\ 8 \qquad (\mathrm{B})\ 20 \qquad (\mathrm{C})\ 24 \qquad (\mathrm{D})\ 27 \qquad (\mathrm{E})\ 36$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 14

Not yet answered

Points out of 6

A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term in the geometric progression?

$(\mathrm{A})\ 1 \qquad (\mathrm{B})\ 4 \qquad (\mathrm{C})\ 36 \qquad (\mathrm{D})\ 49 \qquad (\mathrm{E})\ 81$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 15

Not yet answered

Points out of 6

Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

$(\mathrm{A})\ 250 \qquad (\mathrm{B})\ 300 \qquad (\mathrm{C})\ 350 \qquad (\mathrm{D})\ 400 \qquad (\mathrm{E})\ 500$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 16

Not yet answered

Points out of 6

The set of all real numbers x for which

$$\log_{2004}(\log_{2003}(\log_{2002}(\log_{2001} x)))$$

is defined is $\{x|x > c\}$. What is the value of c?

$(\mathrm{A})0 \qquad (\mathrm{B})2001^{2002} \qquad (\mathrm{C})2002^{2003} \qquad (\mathrm{D})2003^{2004} \qquad (\mathrm{E})2001^{2002^{2003}}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 17

Not yet answered

Points out of 6

Let f be a function with the following properties:

(i) $f(1) = 1$, and

(ii) $f(2n) = n \times f(n)$, for any positive integer n.

What is the value of $f(2^{100})$?

(A) 1 (B) 2^{99} (C) 2^{100} (D) 2^{4950} (E) 2^{9999}

Select one:

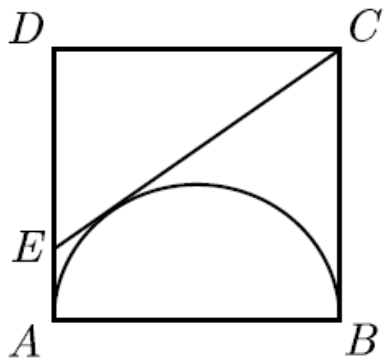
- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 18

Not yet answered

Points out of 6

Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E.



What is the length of $\overline{CE}$?

(A) $\frac{2+\sqrt{5}}{2}$ (B) $\sqrt{5}$ (C) $\sqrt{6}$ (D) $\frac{5}{2}$ (E) $5-\sqrt{5}$

Select one:

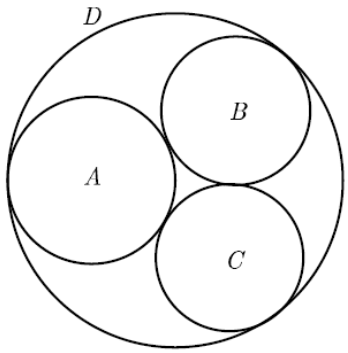
- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 19

Not yet answered

Points out of 6

Circles A, B and C are externally tangent to each other, and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D.



What is the radius of circle B?

$(\mathrm{A})\frac{2}{3}$ $(\mathrm{B})\frac{\sqrt{3}}{2}$ $(\mathrm{C})\frac{7}{8}$ $(\mathrm{D})\frac{8}{9}$ $(\mathrm{E})\frac{1+\sqrt{3}}{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 20

Not yet answered

Points out of 6

Select numbers a and b between 0 and 1 independently and at random, and let c be their sum. Let A, B and C be the results when a, b and c, respectively, are rounded to the nearest integer. What is the probability that $A+B=C$?

$(\mathrm{A})\ \frac{1}{4}$ $(\mathrm{B})\ \frac{1}{3}$ $(\mathrm{C})\ \frac{1}{2}$ $(\mathrm{D})\ \frac{2}{3}$ $(\mathrm{E})\ \frac{3}{4}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 21

Not yet answered

Points out of 6

If $\sum_{n=0}^{\infty} \cos^{2n}\theta = 5$, what is the value of $\cos 2\theta$?

$(\mathrm{A})\frac{1}{5}$ $(\mathrm{B})\frac{2}{5}$ $(\mathrm{C})\frac{\sqrt{5}}{5}$ $(\mathrm{D})\frac{3}{5}$ $(\mathrm{E})\frac{4}{5}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

Three mutually tangent spheres of radius 1 rest on a horizontal plane. A sphere of radius 2 rests on them. What is the distance from the plane to the top of the larger sphere?

$(\mathrm{A})\ 3+\frac{\sqrt{30}}{2}$ $(\mathrm{B})\ 3+\frac{\sqrt{69}}{3}$ $(\mathrm{C})\ 3+\frac{\sqrt{123}}{4}$ $(\mathrm{D})\ \frac{52}{9}$ $(\mathrm{E})\ 3+2\sqrt{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

A polynomial

$$P(x) = c_{2004}x^{2004} + c_{2003}x^{2003} + \ldots + c_1 x + c_0$$

has real coefficients with $c_{2004} \neq 0$ and 2004 distinct complex zeroes $z_k = a_k + b_k i$, $1 \leq k \leq 2004$ with a_k and b_k real, $a_1 = b_1 = 0$, and

$$\sum_{k=1}^{2004} a_k = \sum_{k=1}^{2004} b_k.$$

Which of the following quantities can be a nonzero number?

$(\mathrm{A}) c_0 \qquad (\mathrm{B}) c_{2003} \qquad (\mathrm{C}) b_2 b_3 \ldots b_{2004} \qquad (\mathrm{D}) \sum_{k=1}^{2004} a_k \qquad (\mathrm{E}) \sum_{k=1}^{2004} c_k$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

A plane contains points A and B with $AB = 1$. Let S be the union of all disks of radius 1 in the plane that cover $\overline{AB}$. What is the area of S?

$(\mathrm{A}) 2\pi + \sqrt{3} \qquad (\mathrm{B}) \frac{8\pi}{3} \qquad (\mathrm{C}) 3\pi - \frac{\sqrt{3}}{2} \qquad (\mathrm{D}) \frac{10\pi}{3} - \sqrt{3} \qquad (\mathrm{E}) 4\pi - 2\sqrt{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 25

Not yet answered

Points out of 6

For each integer $n \geq 4$, let a_n denote the base-n number $0.\overline{133}_n$. The product $a_4 a_5 \cdots a_{99}$ can be expressed as $\frac{m}{n!}$, where m and n are positive integers and n is as small as possible. What is the value of m?

$(\mathrm{A})98 \quad (\mathrm{B})101 \quad (\mathrm{C})132 \quad (\mathrm{D})798 \quad (\mathrm{E})962$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**