



# 2005 AMC 12A

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**Question 1**

Not yet answered

Points out of 6

Two is 10% of  $x$  and 20% of  $y$ . What is  $x - y$ ?

- (A) 1      (B) 2      (C) 5      (D) 10      (E) 20

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 2**

Not yet answered

Points out of 6

The equations  $2x + 7 = 3$  and  $bx - 10 = -2$  have the same solution. What is the value of  $b$ ?

- (A)  $-8$       (B)  $-4$       (C)  $2$       (D)  $4$       (E)  $8$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 3**

Not yet answered

Points out of 6

A rectangle with diagonal length  $x$  is twice as long as it is wide. What is the area of the rectangle?

- (A)  $\frac{1}{4}x^2$       (B)  $\frac{2}{5}x^2$       (C)  $\frac{1}{2}x^2$       (D)  $x^2$       (E)  $\frac{3}{2}x^2$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 4**

Not yet answered

Points out of 6

A store normally sells windows at \$100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How much will they save if they purchase the windows together rather than separately?

(A) 100      (B) 200      (C) 300      (D) 400      (E) 500

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 5**

Not yet answered

Points out of 6

The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

(A) 23      (B) 24      (C) 25      (D) 10      (E) 27

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 6**

Not yet answered

Points out of 6

Josh and Mike live 13 miles apart. Yesterday, Josh started to ride his bicycle toward Mike's house. A little later Mike started to ride his bicycle toward Josh's house. When they met, Josh had ridden for twice the length of time as Mike and at four-fifths of Mike's rate. How many miles had Mike ridden when they met?

- (A) 4    (B) 5    (C) 6    (D) 7    (E) 8

Select one:

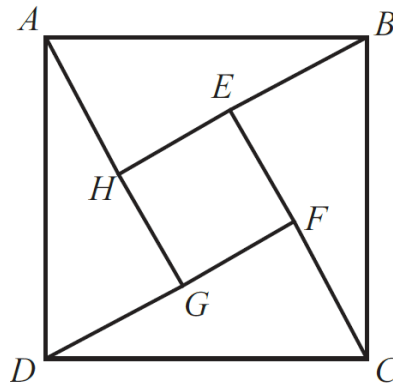
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 7**

Not yet answered

Points out of 6

Square  $EFGH$  is inside the square  $ABCD$  so that each side of  $EFGH$  can be extended to pass through a vertex of  $ABCD$ . Square  $ABCD$  has side length  $\sqrt{50}$  and  $BE = 1$ .



What is the area of the inner square  $EFGH$ ?

- (A) 25    (B) 32    (C) 36    (D) 40    (E) 42

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 8**

Not yet answered

Points out of 6

Let  $A$ ,  $M$ , and  $C$  be digits with

$$(100A + 10M + C)(A + M + C) = 2005$$

What is  $A$ ?

- (A) 1    (B) 2    (C) 3    (D) 4    (E) 5

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 9**

Not yet answered

Points out of 6

There are two values of  $a$  for which the equation  $4x^2 + ax + 8x + 9 = 0$  has only one solution for  $x$ . What is the sum of these values of  $a$ ?

- (A)  $-16$     (B)  $-8$     (C)  $0$     (D)  $8$     (E)  $20$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 10**

Not yet answered

Points out of 6

A wooden cube  $n$  units on a side is painted red on all six faces and then cut into  $n^3$  unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is  $n$ ?

- (A) 3    (B) 4    (C) 5    (D) 6    (E) 7

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 11**

Not yet answered

Points out of 6

How many three-digit numbers satisfy the property that the middle digit is the average of the first and the last digits?

(A) 41      (B) 42      (C) 43      (D) 44      (E) 45

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 12**

Not yet answered

Points out of 6

A line passes through  $A(1, 1)$  and  $B(100, 1000)$ . How many other points with integer coordinates are on the line and strictly between  $A$  and  $B$ ?

(A) 0      (B) 2      (C) 3      (D) 8      (E) 9

Select one:

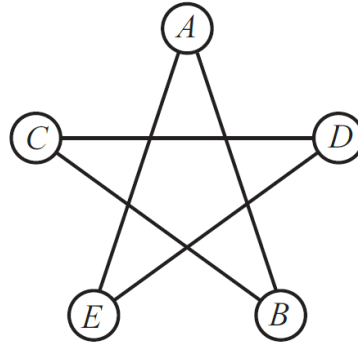
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 13**

Not yet answered

Points out of 6

The regular 5-point star  $ABCDE$  is drawn and in each vertex, there is a number. Each  $A, B, C, D,$  and  $E$  are chosen such that all 5 of them came from set  $\{3, 5, 6, 7, 9\}$ . Each letter is a different number (so one possible way is  $A = 3, B = 5, C = 6, D = 7, E = 9$ ). Let  $AB$  be the sum of the numbers on  $A$  and  $B$ , and so forth. If  $AB, BC, CD, DE,$  and  $EA$  form an arithmetic sequence (not necessarily in increasing order), find the value of  $CD$ .



- (A) 9      (B) 10      (C) 11      (D) 12      (E) 13

Select one:

- A  
 B  
 C  
 D  
 E  
 Leave blank (1.5 points)

**Question 14**

Not yet answered

Points out of 6

On a standard die one of the dots is removed at random with each dot equally likely to be chosen. The die is then rolled. What is the probability that the top face has an odd number of dots?

- (A)  $\frac{5}{11}$       (B)  $\frac{10}{21}$       (C)  $\frac{1}{2}$       (D)  $\frac{11}{21}$       (E)  $\frac{6}{11}$

Select one:

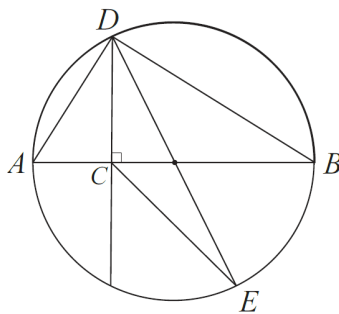
- A  
 B  
 C  
 D  
 E  
 Leave blank (1.5 points)

**Question 15**

Not yet answered

Points out of 6

Let  $\overline{AB}$  be a diameter of a circle and  $C$  be a point on  $\overline{AB}$  with  $2 \cdot AC = BC$ . Let  $D$  and  $E$  be points on the circle such that  $\overline{DC} \perp \overline{AB}$  and  $\overline{DE}$  is a second diameter.



What is the ratio of the area of  $\triangle DCE$  to the area of  $\triangle ABD$ ?

- (A)  $\frac{1}{6}$       (B)  $\frac{1}{4}$       (C)  $\frac{1}{3}$       (D)  $\frac{1}{2}$       (E)  $\frac{2}{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

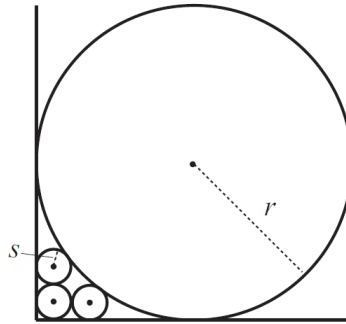


**Question 16**

Not yet answered

Points out of 6

Three circles of radius  $s$  are drawn in the first quadrant of the  $xy$ -plane. The first circle is tangent to both axes, the second is tangent to the first circle and the  $x$ -axis, and the third is tangent to the first circle and the  $y$ -axis. A circle of radius  $r > s$  is tangent to both axes and to the second and third circles.

What is  $\frac{r}{s}$ ?

- (A) 5      (B) 6      (C) 8      (D) 9      (E) 10

Select one:

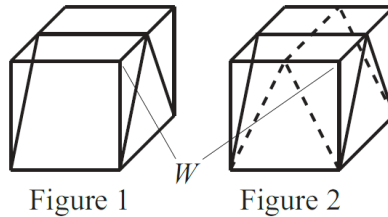
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 17**

Not yet answered

Points out of 6

A unit cube is cut twice to form three triangular prisms, two of which are congruent, as shown in Figure 1. The cube is then cut in the same manner along the dashed lines shown in Figure 2. This creates nine pieces.



What is the volume of the piece that contains vertex  $W$ ?

- (A)  $\frac{1}{12}$     (B)  $\frac{1}{9}$     (C)  $\frac{1}{8}$     (D)  $\frac{1}{6}$     (E)  $\frac{1}{4}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 18**

Not yet answered

Points out of 6

Call a number *prime-looking* if it is composite but not divisible by 2, 3, or 5. The three smallest prime-looking numbers are 49, 77, and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are there less than 1000?

- (A) 100    (B) 102    (C) 104    (D) 106    (E) 108

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 19**

Not yet answered

Points out of 6

A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. If the odometer now reads 002005, how many miles has the car actually traveled? (A) 1404 (B) 1462 (C) 1604 (D) 1605 (E) 1804

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 20**

Not yet answered

Points out of 6

For each  $x$  in  $[0, 1]$ , define

$$f(x) = 2x, \quad \text{if } 0 \leq x \leq \frac{1}{2};$$
$$f(x) = 2 - 2x, \quad \text{if } \frac{1}{2} < x \leq 1.$$

Let  $f^{[2]}(x) = f(f(x))$ , and  $f^{[n+1]}(x) = f^{[n]}(f(x))$  for each integer  $n \geq 2$ . For how many values of  $x$  in  $[0, 1]$  is  $f^{[2005]}(x) = \frac{1}{2}$ ?

(A) 0 (B) 2005 (C) 4010 (D)  $2005^2$  (E)  $2^{2005}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 21**

Not yet answered

Points out of 6

How many ordered triples of integers  $(a, b, c)$ , with  $a \geq 2$ ,  $b \geq 1$ , and  $c \geq 0$ , satisfy both  $\log_a b = c^{2005}$  and  $a + b + c = 2005$ ?

- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 22**

Not yet answered

Points out of 6

A rectangular box  $P$  is inscribed in a sphere of radius  $r$ . The surface area of  $P$  is 384, and the sum of the lengths of its 12 edges is 112. What is  $r$ ?

- (A) 8      (B) 10      (C) 12      (D) 14      (E) 16

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 23**

Not yet answered

Points out of 6

Two distinct numbers  $a$  and  $b$  are chosen randomly from the set  $\{2, 2^2, 2^3, \dots, 2^{25}\}$ . What is the probability that  $\log_a b$  is an integer?

- (A)  $\frac{2}{25}$       (B)  $\frac{31}{300}$       (C)  $\frac{13}{100}$       (D)  $\frac{7}{50}$       (E)  $\frac{1}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 24**

Not yet answered

Points out of 6

Let  $P(x) = (x - 1)(x - 2)(x - 3)$ . For how many polynomials  $Q(x)$  does there exist a polynomial  $R(x)$  of degree 3 such that  $P(Q(x)) = P(x) * R(x)$ ?

(A)19      (B)22      (C)24      (D)27      (E)32

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 25**

Not yet answered

Points out of 6

Let  $S$  be the set of all points with coordinates  $(x, y, z)$ , where  $x, y,$  and  $z$  are each chosen from the set  $\{0, 1, 2\}$ . How many equilateral triangles all have their vertices in  $S$ ?

(A) 72      (B) 76      (C) 80      (D) 84      (E) 88

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)