



2005 AMC 12B

For more practice and resources, visit ziml.aretteam.org

The problems in the AMC-Series Contests are copyrighted by American Mathematics Competitions at Mathematical Association of America (www.maa.org).



Question 1

Not yet answered

Points out of 6

A scout troop buys 1000 candy bars at a price of five for 2 dollars. They sell all the candy bars at the price of two for 1 dollar. What was their profit, in dollars?

- (A) 100 (B) 200 (C) 300 (D) 400 (E) 500

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 2

Not yet answered

Points out of 6

A positive number x has the property that $x\%$ of x is 4. What is x ?

- (A) 2 (B) 4 (C) 10 (D) 20 (E) 40

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 3

Not yet answered

Points out of 6

Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

- (A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{2}{3}$ (E) $\frac{4}{5}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

At the beginning of the school year, Lisa's goal was to earn an A on at least 80% of her 50 quizzes for the year. She earned an A on 22 of the first 30 quizzes. If she is to achieve her goal, on at most how many of the remaining quizzes can she earn a grade lower than an A?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Select one:

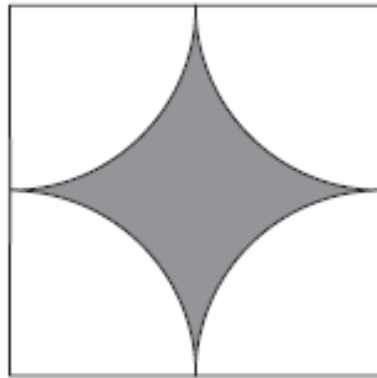
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

An 8-foot by 10-foot floor is tiled with square tiles of size 1 foot by 1 foot. Each tile has a pattern consisting of four white quarter circles of radius $1/2$ foot centered at each corner of the tile. The remaining portion of the tile is shaded.



How many square feet of the floor are shaded?

- (A) $80 - 20\pi$ (B) $60 - 10\pi$ (C) $80 - 10\pi$ (D) $60 + 10\pi$ (E) $80 + 10\pi$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

In $\triangle ABC$, we have $AC = BC = 7$ and $AB = 2$. Suppose that D is a point on line AB such that B lies between A and D and $CD = 8$. What is BD ?

- (A) 3 (B) $2\sqrt{3}$ (C) 4 (D) 5 (E) $4\sqrt{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

What is the area enclosed by the graph of $|3x| + |4y| = 12$?

- (A) 6 (B) 12 (C) 16 (D) 24 (E) 25

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

For how many values of a is it true that the line $y = x + a$ passes through the vertex of the parabola $y = x^2 + a^2$?

- (A) 0 (B) 1 (C) 2 (D) 10 (E) infinitely many

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

- (A) 0 (B) 1 (C) 2 (D) 4 (E) 5

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

The first term of a sequence is 2005. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2005th term of the sequence?

- (A) 29 (B) 55 (C) 85 (D) 133 (E) 250

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

- (A) $\frac{1}{4}$ (B) $\frac{2}{5}$ (C) $\frac{3}{7}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of m , n , and p is zero. What is the value of n/p ?

- (A) 1 (B) 2 (C) 4 (D) 8 (E) 16

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

Suppose that $4^{x_1} = 5$, $5^{x_2} = 6$, $6^{x_3} = 7$, ..., $127^{x_{124}} = 128$. What is $x_1 x_2 \dots x_{124}$?

- (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

A circle having center $(0, k)$, with $k > 6$, is tangent to the lines $y = x$, $y = -x$ and $y = 6$. What is the radius of this circle?

- (A) $6\sqrt{2} - 6$ (B) 6 (C) $6\sqrt{2}$ (D) 12 (E) $6 + 6\sqrt{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 15

Not yet answered

Points out of 6

The sum of four two-digit numbers is 221. None of the eight digits is 0 and no two of them are the same. Which of the following is **not** included among the eight digits?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

Eight spheres of radius 1, one per octant, are each tangent to the coordinate planes. What is the radius of the smallest sphere, centered at the origin, that contains these eight spheres?

- (A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $1 + \sqrt{2}$ (D) $1 + \sqrt{3}$ (E) 3

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 17

Not yet answered

Points out of 6

How many distinct four-tuples (a, b, c, d) of rational numbers are there with

$$a \cdot \log_{10} 2 + b \cdot \log_{10} 3 + c \cdot \log_{10} 5 + d \cdot \log_{10} 7 = 2005?$$

- (A) 0 (B) 1 (C) 17 (D) 2004 (E) infinitely many

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

Let $A(2, 2)$ and $B(7, 7)$ be points in the plane. Define R as the region in the first quadrant consisting of those points C such that $\triangle ABC$ is an acute triangle. What is the closest integer to the area of the region R ?

- (A) 25 (B) 39 (C) 51 (D) 60 (E) 80

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

Let x and y be two-digit integers such that y is obtained by reversing the digits of x . The integers x and y satisfy $x^2 - y^2 = m^2$ for some positive integer m . What is $x + y + m$?

- (A) 88 (B) 112 (C) 116 (D) 144 (E) 154

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

Let a, b, c, d, e, f, g and h be distinct elements in the set $\{-7, -5, -3, -2, 2, 4, 6, 13\}$.

What is the minimum possible value of $(a + b + c + d)^2 + (e + f + g + h)^2$?

- (A) 30 (B) 32 (C) 34 (D) 40 (E) 50

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

A positive integer n has 60 divisors and $7n$ has 80 divisors. What is the greatest integer k such that 7^k divides n ?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 22

Not yet answered

Points out of 6

A sequence of complex numbers z_0, z_1, z_2, \dots is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}},$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many possible values are there for z_0 ?

- (A) 1 (B) 2 (C) 4 (D) 2005 (E) 2^{2005}

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

Let S be the set of ordered triples (x, y, z) of real numbers for which

$$\log_{10}(x + y) = z \text{ and } \log_{10}(x^2 + y^2) = z + 1.$$

There are real numbers a and b such that for all ordered triples (x, y, z) in S we have $x^3 + y^3 = a \cdot 10^{3z} + b \cdot 10^{2z}$. What is the value of $a + b$?

- (A) $\frac{15}{2}$ (B) $\frac{29}{2}$ (C) 15 (D) $\frac{39}{2}$ (E) 24

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

All three vertices of an equilateral triangle are on the parabola $y = x^2$, and one of its sides has a slope of 2. The x -coordinates of the three vertices have a sum of m/n , where m and n are relatively prime positive integers. What is the value of $m + n$?

- (A) 14 (B) 15 (C) 16 (D) 17 (E) 18

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

Six ants simultaneously stand on the six vertices of a regular octahedron, with each ant at a different vertex. Simultaneously and independently, each ant moves from its vertex to one of the four adjacent vertices, each with equal probability. What is the probability that no two ants arrive at the same vertex?

- (A) $\frac{5}{256}$ (B) $\frac{21}{1024}$ (C) $\frac{11}{512}$ (D) $\frac{23}{1024}$ (E) $\frac{3}{128}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)