



2008 AMC 12A

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Question 1

Not yet answered

Points out of 6

A bakery owner turns on his doughnut machine at 8:30 AM. At 11:10 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the job?

- (A) 1:50 PM (B) 3:00 PM (C) 3:30 PM (D) 4:30 PM (E) 5:50 PM

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 2

Not yet answered

Points out of 6

What is the reciprocal of $\frac{1}{2} + \frac{2}{3}$?

- (A) $\frac{6}{7}$ (B) $\frac{7}{6}$ (C) $\frac{5}{3}$ (D) 3 (E) $\frac{7}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 3

Not yet answered

Points out of 6

Suppose that $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\frac{1}{2}$ of 5 bananas?

- (A) 2 (B) $\frac{5}{2}$ (C) 3 (D) $\frac{7}{2}$ (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

Which of the following is equal to the product

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdots \frac{4n+4}{4n} \cdots \frac{2008}{2004}?$$

- (A) 251 (B) 502 (C) 1004 (D) 2008 (E) 4016

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

Suppose that

$$\frac{2x}{3} - \frac{x}{6}$$

is an integer. Which of the following statements must be true about x ?

- (A) It is negative.
- (B) It is even, but not necessarily a multiple of 3.
- (C) It is a multiple of 3, but not necessarily even.
- (D) It is a multiple of 6, but not necessarily a multiple of 12.
- (E) It is a multiple of 12.

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

Heather compares the price of a new computer at two different stores. Store A offers 15% off the sticker price followed by a \$90 rebate, and store B offers 25% off the same sticker price with no rebate. Heather saves \$15 by buying the computer at store A instead of store B . What is the sticker price of the computer, in dollars?

- (A) 750 (B) 900 (C) 1000 (D) 1050 (E) 1500

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

While Steve and LeRoy are fishing 1 mile from shore, their boat springs a leak, and water comes in at a constant rate of 10 gallons per minute. The boat will sink if it takes in more than 30 gallons of water. Steve starts rowing toward the shore at a constant rate of 4 miles per hour while LeRoy bails water out of the boat. What is the slowest rate, in gallons per minute, at which LeRoy can bail if they are to reach the shore without sinking?

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

What is the volume of a cube whose surface area is twice that of a cube with volume 1?

(A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4 (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

Older television screens have an aspect ratio of 4 : 3. That is, the ratio of the width to the height is 4 : 3. The aspect ratio of many movies is not 4 : 3, so they are sometimes shown on a television screen by "letterboxing" - darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of 2 : 1 and is shown on an older television screen with a 27-inch diagonal.



What is the height, in inches, of each darkened strip?

- (A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t ?

- (A) $\left(\frac{1}{5} + \frac{1}{7}\right)(t + 1) = 1$ (B) $\left(\frac{1}{5} + \frac{1}{7}\right)t + 1 = 1$ (C) $\left(\frac{1}{5} + \frac{1}{7}\right)t = 1$
- (D) $\left(\frac{1}{5} + \frac{1}{7}\right)(t - 1) = 1$ (E) $(5 + 7)t = 1$

Select one:

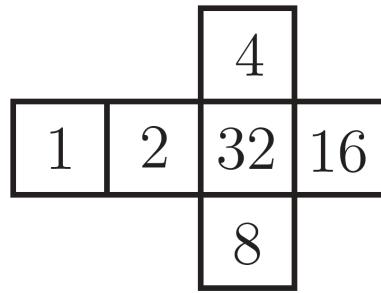
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

Three cubes are each formed from the pattern shown. They are then stacked on a table one on top of another so that the 13 visible numbers have the greatest possible sum.



What is that sum?

- (A) 154 (B) 159 (C) 164 (D) 167 (E) 189

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

A function f has domain $[0, 2]$ and range $[0, 1]$. (The notation $[a, b]$ denotes $\{x : a \leq x \leq b\}$.) What are the domain and range, respectively, of the function g defined by $g(x) = 1 - f(x + 1)$?

- (A) $[-1, 1], [-1, 0]$ (B) $[-1, 1], [0, 1]$ (C) $[0, 2], [-1, 0]$
(D) $[1, 3], [-1, 0]$ (E) $[1, 3], [0, 1]$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

Points A and B lie on a circle centered at O , and $\angle AOB = 60^\circ$. A second circle is internally tangent to the first and tangent to both \overline{OA} and \overline{OB} . What is the ratio of the area of the smaller circle to that of the larger circle?

- (A) $\frac{1}{16}$ (B) $\frac{1}{9}$ (C) $\frac{1}{8}$ (D) $\frac{1}{6}$ (E) $\frac{1}{4}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

What is the area of the region defined by the inequality $|3x - 18| + |2y + 7| \leq 3$?

- (A) 3 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$ (E) 5

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 15

Not yet answered

Points out of 6

Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

- (A) 0 (B) 2 (C) 4 (D) 6 (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

The numbers $\log(a^3b^7)$, $\log(a^5b^{12})$, and $\log(a^8b^{15})$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log b^n$. What is n ?

(A) 40 (B) 56 (C) 76 (D) 112 (E) 143

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 17

Not yet answered

Points out of 6

Let a_1, a_2, \dots be a sequence determined by the rule $a_n = a_{n-1}/2$ if a_{n-1} is even and $a_n = 3a_{n-1} + 1$ if a_{n-1} is odd. For how many positive integers $a_1 \leq 2008$ is it true that a_1 is less than each of a_2, a_3 , and a_4 ?

(A) 250 (B) 251 (C) 501 (D) 502 (E) 1004

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

Triangle ABC , with sides of length 5, 6, and 7, has one vertex on the positive x -axis, one on the positive y -axis, and one on the positive z -axis. Let O be the origin. What is the volume of tetrahedron $OABC$?

(A) $\sqrt{85}$ (B) $\sqrt{90}$ (C) $\sqrt{95}$ (D) 10 (E) $\sqrt{105}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

In the expansion of

$$(1 + x + x^2 + \cdots + x^{27}) (1 + x + x^2 + \cdots + x^{14})^2,$$

what is the coefficient of x^{28} ?

- (A) 195 (B) 196 (C) 224 (D) 378 (E) 405

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

Triangle ABC has $AC = 3$, $BC = 4$, and $AB = 5$. Point D is on \overline{AB} , and \overline{CD} bisects the right angle. The inscribed circles of $\triangle ADC$ and $\triangle BCD$ have radii r_a and r_b , respectively. What is r_a/r_b ?

- (A) $\frac{1}{28}(10 - \sqrt{2})$
- (B) $\frac{3}{56}(10 - \sqrt{2})$
- (C) $\frac{1}{14}(10 - \sqrt{2})$
- (D) $\frac{5}{56}(10 - \sqrt{2})$
- (E) $\frac{3}{28}(10 - \sqrt{2})$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is heavy-tailed if $a_1 + a_2 < a_4 + a_5$.
What is the number of heavy-tailed permutations?

- (A) 36 (B) 40 (C) 44 (D) 48 (E) 52

Select one:

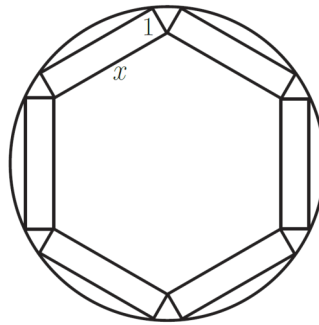
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 22

Not yet answered

Points out of 6

A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x . Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat.

What is x ?

- (A) $2\sqrt{5} - \sqrt{3}$ (B) 3 (C) $\frac{3\sqrt{7} - \sqrt{3}}{2}$ (D) $2\sqrt{3}$ (E) $\frac{5 + 2\sqrt{3}}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

The solutions of the equation $z^4 + 4z^3i - 6z^2 - 4zi - i = 0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

- (A) $2^{5/8}$ (B) $2^{3/4}$ (C) 2 (D) $2^{5/4}$ (E) $2^{3/2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

Triangle ABC has $\angle C = 60^\circ$ and $BC = 4$. Point D is the midpoint of BC . What is the largest possible value of $\tan \angle BAD$?

- (A) $\frac{\sqrt{3}}{6}$ (B) $\frac{\sqrt{3}}{3}$ (C) $\frac{\sqrt{3}}{2\sqrt{2}}$ (D) $\frac{\sqrt{3}}{4\sqrt{2}-3}$ (E) 1

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

A sequence $(a_1, b_1), (a_2, b_2), (a_3, b_3), \dots$ of points in the coordinate plane satisfies

$$(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n) \text{ for } n = 1, 2, 3, \dots$$

Suppose that $(a_{100}, b_{100}) = (2, 4)$. What is $a_1 + b_1$?

(A) $-\frac{1}{2^{97}}$ (B) $-\frac{1}{2^{99}}$ (C) 0 (D) $\frac{1}{2^{98}}$ (E) $\frac{1}{2^{96}}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)