



# 2009 AMC 12A

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**Question 1**

Not yet answered

Points out of 6

Kim's flight took off from Newark at 10:34 AM and landed in Miami at 1:18 PM. Both cities are in the same time zone. If her flight took  $h$  hours and  $m$  minutes, with  $0 < m < 60$ , what is  $h + m$ ?

(A) 46      (B) 47      (C) 50      (D) 53      (E) 54

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 2**

Not yet answered

Points out of 6

Which of the following is equal to  $1 + \frac{1}{1 + \frac{1}{1+1}}$ ?

(A)  $\frac{5}{4}$       (B)  $\frac{3}{2}$       (C)  $\frac{5}{3}$       (D) 2      (E) 3

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 3**

Not yet answered

Points out of 6

What number is one third of the way from  $\frac{1}{4}$  to  $\frac{3}{4}$ ?

- (A)  $\frac{1}{3}$     (B)  $\frac{5}{12}$     (C)  $\frac{1}{2}$     (D)  $\frac{7}{12}$     (E)  $\frac{2}{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 4**

Not yet answered

Points out of 6

Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could *not* be the total value of the four coins, in cents?

- (A) 15    (B) 25    (C) 35    (D) 45    (E) 55

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 5**

Not yet answered

Points out of 6

One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

- (A) 8    (B) 27    (C) 64    (D) 125    (E) 216

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 6**

Not yet answered

Points out of 6

Suppose that  $P = 2^m$  and  $Q = 3^n$ . Which of the following is equal to  $12^{mn}$  for every pair of integers  $(m, n)$ ?

- (A)  $P^2Q$       (B)  $P^nQ^m$       (C)  $P^nQ^{2m}$       (D)  $P^{2m}Q^n$       (E)  $P^{2n}Q^m$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 7**

Not yet answered

Points out of 6

The first three terms of an arithmetic sequence are  $2x - 3$ ,  $5x - 11$ , and  $3x + 1$  respectively. The  $n$ th term of the sequence is 2009. What is  $n$ ?

- (A) 255      (B) 502      (C) 1004      (D) 1506      (E) 8037

Select one:

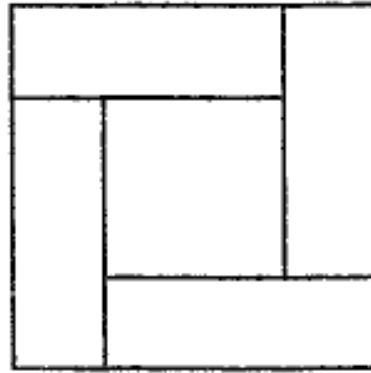
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 8**

Not yet answered

Points out of 6

Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square.



What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?

- (A) 3      (B)  $\sqrt{10}$       (C)  $2 + \sqrt{2}$       (D)  $2\sqrt{3}$       (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 9**

Not yet answered

Points out of 6

Suppose that  $f(x + 3) = 3x^2 + 7x + 4$  and  $f(x) = ax^2 + bx + c$ . What is  $a + b + c$ ?

- (A) -1      (B) 0      (C) 1      (D) 2      (E) 3

Select one:

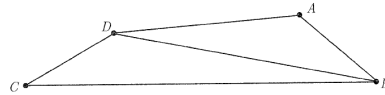
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 10**

Not yet answered

Points out of 6

In quadrilateral  $ABCD$ ,  $AB = 5$ ,  $BC = 17$ ,  $CD = 5$ ,  $DA = 9$ , and  $BD$  is an integer.



What is  $BD$ ?

- (A) 11      (B) 12      (C) 13      (D) 14      (E) 15

Select one:

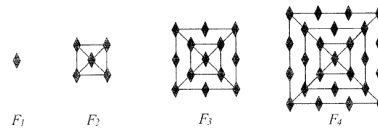
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 11**

Not yet answered

Points out of 6

The figures  $F_1$ ,  $F_2$ ,  $F_3$ , and  $F_4$  shown are the first in a sequence of figures. For  $n \geq 3$ ,  $F_n$  is constructed from  $F_{n-1}$  by surrounding it with a square and placing one more diamond on each side of the new square than  $F_{n-1}$  had on each side of its outside square. For example, figure  $F_3$  has 13 diamonds.



How many diamonds are there in figure  $F_{20}$ ?

- (A) 401      (B) 485      (C) 585      (D) 626      (E) 761

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 12**

Not yet answered

Points out of 6

How many positive integers less than 1000 are 6 times the sum of their digits?

(A) 0    (B) 1    (C) 2    (D) 4    (E) 12

Select one:

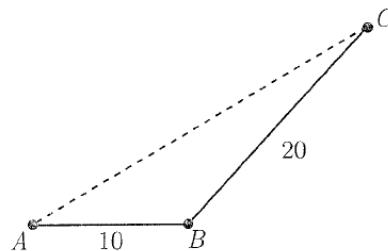
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 13**

Not yet answered

Points out of 6

A ship sails 10 miles in a straight line from  $A$  to  $B$ , turns through an angle between  $45^\circ$  and  $60^\circ$ , and then sails another 20 miles to  $C$ . Let  $AC$  be measured in miles.



Which of the following intervals contains  $AC^2$ ?

(A) [400, 500]    (B) [500, 600]    (C) [600, 700]    (D) [700, 800]  
(E) [800, 900]

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 14**

Not yet answered

Points out of 6

A triangle has vertices  $(0, 0)$ ,  $(1, 1)$ , and  $(6m, 0)$ , and the line  $y = mx$  divides the triangle into two triangles of equal area. What is the sum of all possible values of  $m$ ?

- (A)**  $-\frac{1}{3}$     **(B)**  $-\frac{1}{6}$     **(C)**  $\frac{1}{6}$     **(D)**  $\frac{1}{3}$     **(E)**  $\frac{1}{2}$

Select one:

- A**
- B**
- C**
- D**
- E**
- Leave blank (1.5 points)**

**Question 15**

Not yet answered

Points out of 6

For what value of  $n$  is  $i + 2i^2 + 3i^3 + \dots + ni^n = 48 + 49i$ ?

Note: here  $i = \sqrt{-1}$ .

- (A)** 24    **(B)** 48    **(C)** 49    **(D)** 97    **(E)** 98

Select one:

- A**
- B**
- C**
- D**
- E**
- Leave blank (1.5 points)**

**Question 16**

Not yet answered

Points out of 6

A circle with center  $C$  is tangent to the positive  $x$  and  $y$ -axes and externally tangent to the circle centered at  $(3, 0)$  with radius 1. What is the sum of all possible radii of the circle with center  $C$ ?

- (A)** 3    **(B)** 4    **(C)** 6    **(D)** 8    **(E)** 9

Select one:

- A**
- B**
- C**
- D**
- E**
- Leave blank (1.5 points)**



**Question 17**

Not yet answered

Points out of 6

Let  $a + ar_1 + ar_1^2 + ar_1^3 + \dots$  and  $a + ar_2 + ar_2^2 + ar_2^3 + \dots$  be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is  $r_1$ , and the sum of the second series is  $r_2$ . What is  $r_1 + r_2$ ?

- (A) 0      (B)  $\frac{1}{2}$       (C) 1      (D)  $\frac{1 + \sqrt{5}}{2}$       (E) 2

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 18**

Not yet answered

Points out of 6

For  $k > 0$ , let  $I_k = 10 \dots 064$ , where there are  $k$  zeros between the 1 and the 6. Let  $N(k)$  be the number of factors of 2 in the prime factorization of  $I_k$ . What is the maximum value of  $N(k)$ ?

- (A) 6      (B) 7      (C) 8      (D) 9      (E) 10

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 19**

Not yet answered

Points out of 6

Andrea inscribed a circle inside a regular pentagon, circumscribed a circle around the pentagon, and calculated the area of the region between the two circles. Bethany did the same with a regular heptagon (7 sides). The areas of the two regions were  $A$  and  $B$ , respectively. Each polygon had a side length of 2. Which of the following is true?

(A)  $A = \frac{25}{49}B$     (B)  $A = \frac{5}{7}B$     (C)  $A = B$     (D)  $A = \frac{7}{5}B$     (E)  $A = \frac{49}{25}B$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 20**

Not yet answered

Points out of 6

Convex quadrilateral  $ABCD$  has  $AB = 9$  and  $CD = 12$ . Diagonals  $AC$  and  $BD$  intersect at  $E$ ,  $AC = 14$ , and  $\triangle AED$  and  $\triangle BEC$  have equal areas. What is  $AE$ ?

(A)  $\frac{9}{2}$     (B)  $\frac{50}{11}$     (C)  $\frac{21}{4}$     (D)  $\frac{17}{3}$     (E) 6

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 21**

Not yet answered

Points out of 6

Let  $p(x) = x^3 + ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are complex numbers. Suppose that

What is the number of nonreal zeros of  $x^{12} + ax^8 + bx^4 + c$ ?

(A) 4      (B) 6      (C) 8      (D) 10      (E) 12

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 22**

Not yet answered

Points out of 6

A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into the two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area  $\frac{a\sqrt{b}}{c}$ , where  $a$ ,  $b$ , and  $c$  are positive integers,  $a$  and  $c$  are relatively prime, and  $b$  is not divisible by the square of any prime. What is  $a + b + c$ ?

(A) 10      (B) 11      (C) 12      (D) 13      (E) 14

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 23**

Not yet answered

Points out of 6

Functions  $f$  and  $g$  are quadratic,  $g(x) = -f(100 - x)$ , and the graph of  $g$  contains the vertex of the graph of  $f$ . The four  $x$ -intercepts on the two graphs have  $x$ -coordinates  $x_1$ ,  $x_2$ ,  $x_3$ , and  $x_4$ , in increasing order, and  $x_3 - x_2 = 150$ . The value of  $x_4 - x_1$  is  $m + n\sqrt{p}$ , where  $m$ ,  $n$ , and  $p$  are positive integers, and  $p$  is not divisible by the square of any prime. What is  $m + n + p$ ?

- (A) 602      (B) 652      (C) 702      (D) 752      (E) 802

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 24**

Not yet answered

Points out of 6

The *tower function of twos* is defined recursively as follows:  $T(1) = 2$  and  $T(n + 1) = 2^{T(n)}$  for  $n \geq 1$ . Let  $A = (T(2009))^{T(2009)}$  and  $B = (T(2009))^A$ . What is the largest integer  $k$  such that

$$\underbrace{\log_2 \log_2 \log_2 \dots \log_2 B}_{k \text{ times}}$$

is defined?

- (A) 2009      (B) 2010      (C) 2011      (D) 2012      (E) 2013

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

**Question 25**

Not yet answered

Points out of 6

The first two terms of a sequence are  $a_1 = 1$  and  $a_2 = \frac{1}{\sqrt{3}}$ . For  $n \geq 1$ ,

$$a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

What is  $|a_{2009}|$ ?

- (A) 0      (B)  $2 - \sqrt{3}$       (C)  $\frac{1}{\sqrt{3}}$       (D) 1      (E)  $2 + \sqrt{3}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)