



2010 AMC 12A

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Question 1

Not yet answered

Points out of 6

What is $(20 - (2010 - 201)) + (2010 - (201 - 20))$?**(A)** - 4020 **(B)** 0 **(C)** 40 **(D)** 401 **(E)** 4020

Select one:

- A**
- B**
- C**
- D**
- E**
- Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

A ferry boat shuttles tourists to an island every hour starting at 10 AM until its last trip, which starts at 3 PM. One day the boat captain notes that on the 10 AM trip there were 100 tourists on the ferry boat, and that on each successive trip, the number of tourists was 1 fewer than on the previous trip. How many tourists did the ferry take to the island that day?

(A) 585 **(B)** 594 **(C)** 672 **(D)** 679 **(E)** 694

Select one:

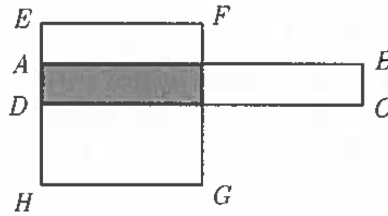
- A**
- B**
- C**
- D**
- E**
- Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

Rectangle $ABCD$, pictured below, shares 50% of its area with square $EFGH$. Square $EFGH$ shares 20% of its area with rectangle $ABCD$.



What is $\frac{AB}{AD}$?

- (A) 4 (B) 5 (C) 6 (D) 8 (E) 10

Select one:

- A
 B
 C
 D
 E
 Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

If $x < 0$, then which of the following must be positive?

- (A) $\frac{x}{|x|}$ (B) $-x^2$ (C) -2^x (D) $-x^{-1}$ (E) $\sqrt[3]{x}$

Select one:

- A
 B
 C
 D
 E
 Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

Halfway through a 100-shot archery tournament, Chelsea leads by 50 points. For each shot a bullseye scores 10 points, with other possible scores being 8, 4, 2, and 0 points. Chelsea always scores at least 4 points on each shot. If Chelsea's next n shots are bullseyes she will be guaranteed victory. What is the minimum value for n ?

(A) 38 (B) 40 (C) 42 (D) 44 (E) 46

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

A palindrome, such as 83438, is a number that remains the same when its digits are reversed. The numbers x and $x + 32$ are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x ?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

Logan is constructing a scaled model of his town. The city's water tower stands 40 meters high, and the top portion is a sphere that holds 100,000 liters of water. Logan's miniature water tower holds 0.1 liters. How tall, in meters, should Logan make his tower?

- (A) 0.04 (B) $\frac{0.4}{\pi}$ (C) 0.4 (D) $\frac{4}{\pi}$ (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on \overline{AB} and \overline{BC} , respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD , and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

- (A) 60° (B) 75° (C) 90° (D) 105° (E) 120°

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

- (A) 7 (B) 8 (C) 10 (D) 12 (E) 15

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

The first four terms of an arithmetic sequence are p , 9, $3p - q$, and $3p + q$. What is the 2010th term of this sequence?

- (A) 8041 (B) 8043 (C) 8045 (D) 8047 (E) 8049

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b 7^7$. What is b ?

- (A) $\frac{7}{15}$ (B) $\frac{7}{8}$ (C) $\frac{8}{7}$ (D) $\frac{15}{8}$ (E) $\frac{15}{7}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

For how many integer values of k do the graphs of $x^2 + y^2 = k^2$ and $xy = k$ not intersect?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

Nondegenerate $\triangle ABC$ has integer side lengths, \overline{BD} is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?

- (A) 30 (B) 33 (C) 35 (D) 36 (E) 37

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 15

Not yet answered

Points out of 6

A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

- (A) $\frac{\sqrt{15} - 3}{6}$
- (B) $\frac{6 - \sqrt{6\sqrt{6} + 2}}{12}$
- (C) $\frac{\sqrt{2} - 1}{2}$
- (D) $\frac{3 - \sqrt{3}}{6}$
- (E) $\frac{\sqrt{3} - 1}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

Bernardo randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

- (A) $\frac{47}{72}$ (B) $\frac{37}{56}$ (C) $\frac{2}{3}$ (D) $\frac{49}{72}$ (E) $\frac{39}{56}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 17

Not yet answered

Points out of 6

Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r ?

- (A) $\frac{4\sqrt{3}}{3}$ (B) $\frac{10}{3}$ (C) 4 (D) $\frac{17}{4}$ (E) 6

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

A 16-step path is to go from $(-4, -4)$ to $(4, 4)$ with each step increasing either the x -coordinate or the y -coordinate by 1. How many such paths stay outside or on the boundary of the square $-2 \leq x \leq 2, -2 \leq y \leq 2$ at each step?

(A) 92 (B) 144 (C) 1568 (D) 1698 (E) 12, 800

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

Each of 2010 boxes in a line contains a single red marble, and for $1 \leq k \leq 2010$, the box in the k th position also contains k white marbles. Isabella begins at the first box and successively draws a single marble at random from each box, in order. She stops when she first draws a red marble. Let $P(n)$ be the probability that Isabella stops after drawing exactly n marbles. What is the smallest value of n for which $P(n) < \frac{1}{2010}$?

(A) 45 (B) 63 (C) 64 (D) 201 (E) 1005

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

Arithmetic sequences (a_n) and (b_n) have integer terms with $a_1 = b_1 = 1 < a_2 \leq b_2$ and $a_n b_n = 2010$ for some n . What is the largest possible value of n ?

(A) 2 (B) 3 (C) 8 (D) 288 (E) 2009

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

The graph of $y = x^6 - 10x^5 + 29x^4 - 4x^3 + ax^2$ lies above the line $y = bx + c$ except at three values of x , where the graph and the line intersect. What is the largest of these values?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 22

Not yet answered

Points out of 6

What is the minimum value of

$$f(x) = |x - 1| + |2x - 1| + |3x - 1| + \cdots + |119x - 1|?$$

(A) 49 (B) 50 (C) 51 (D) 52 (E) 53

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

The number obtained from the last two nonzero digits of $90!$ is equal to n . What is n ?

(A) 12 (B) 32 (C) 48 (D) 52 (E) 68

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

Let $f(x) = \log_{10}(\sin(\pi x) \cdot \sin(2\pi x) \cdot \sin(3\pi x) \cdots \sin(8\pi x))$. The intersection of the domain of $f(x)$ with the interval $[0, 1]$ is a union of n disjoint open intervals. What is n ?

(A) 2 (B) 12 (C) 18 (D) 22 (E) 36

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

Two quadrilaterals are considered the same if one can be obtained from the other by a rotation and a translation. How many different convex cyclic quadrilaterals are there with integer sides and perimeter equal to 32?

(A) 560 (B) 564 (C) 568 (D) 1498 (E) 2255

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)