2012 AMC 12B

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Question 1

Not yet answered

Points out of 6

Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

$\textbf{(A)}\ 48 \qquad \textbf{(B)}\ 56 \qquad \textbf{(C)}\ 64 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 80$

Select one:

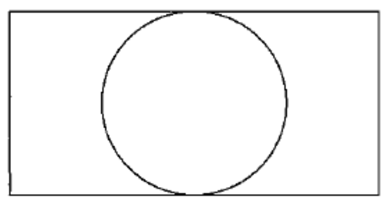
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

A circle of radius 5 is inscribed in a rectangle as shown. The ratio of the length of the rectangle to its width is 2:1.



What is the area of the rectangle?

$\textbf{(A)}\ 50 \qquad \textbf{(B)}\ 100 \qquad \textbf{(C)}\ 125 \qquad \textbf{(D)}\ 150 \qquad \textbf{(E)}\ 200$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

For a science project, Sammy observed a chipmunk and squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 54

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

Suppose that the euro is worth 1.3 dollars. If Diana has 500 dollars and Etienne has 400 euros, by what percent is the value of Etienne's money greater that the value of Diana's money?

(A) 2 **(B)** 4 **(C)** 6.5 **(D)** 8 **(E)** 13

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

Two integers have a sum of 26. when two more integers are added to the first two, the sum is 41. Finally, when two more integers are added to the sum of the previous 4 integers, the sum is 57. What is the minimum number of even integers among the 6 integers?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **6**

Not yet answered

Points out of 6

In order to estimate the value of $x - y$ where x and y are real numbers with $x > y > 0$, Xiaoli rounded x up by a small amount, rounded y down by the same amount, and then subtracted her rounded values. Which of the following statements is necessarily correct?

(A) Her estimate is larger than $x - y$.

(B) Her estimate is smaller than $x - y$.

(C) Her estimate equals $x - y$.

(D) Her estimate equals $y - x$.

(E) Her estimate is 0.

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **7**

Not yet answered

Points out of 6

Small lights are hung on a string 6 inches apart in the order red, red, green, green, green, red, red, green, green, green, and so on continuing this pattern of 2 red lights followed by 3 green lights. How many feet separate the 3rd red light and the 21st red light?

Note: 1 foot is equal to 12 inches.

(A) 18 **(B)** 18.5 **(C)** 20 **(D)** 20.5 **(E)** 22.5

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **8**

Not yet answered

Points out of 6

A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

$\textbf{(A)}\ 729 \qquad \textbf{(B)}\ 972 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 2187 \qquad \textbf{(E)}\ 2304$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **9**

Not yet answered

Points out of 6

It takes Clea 60 seconds to walk down an escalator when it is not moving, and 24 seconds when it is moving. How seconds would it take Clea to ride the escalator down when she is not walking?

$\textbf{(A)}\ 36 \qquad \textbf{(B)}\ 40 \qquad \textbf{(C)}\ 42 \qquad \textbf{(D)}\ 48 \qquad \textbf{(E)}\ 52$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **10**

Not yet answered

Points out of 6

What is the area of the polygon whose vertices are the points of intersection of the curves $x^2+y^2=25$ and $(x-4)^2+9y^2=81?$

$\textbf{(A)}\ 24 \qquad \textbf{(B)}\ 27 \qquad \textbf{(C)}\ 36 \qquad \textbf{(D)}\ 37.5 \qquad \textbf{(E)}\ 42$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 11

Not yet answered

Points out of 6

In the equation below, A and B are consecutive positive integers, and A, B, and $A+B$ represent number bases:

$$132_A + 43_B = 69_{A+B}.$$

What is $A+B$?

(A) 9 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 17

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 12

Not yet answered

Points out of 6

How many sequences of zeros and ones of length 20 have all the zeros consecutive, or all the ones consecutive, or both?

(A) 190 **(B)** 192 **(C)** 211 **(D)** 380 **(E)** 382

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 13

Not yet answered

Points out of 6

Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers, each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas will have a least one point in common?

(A) $\frac{1}{2}$ **(B)** $\frac{25}{36}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{31}{36}$ **(E)** 1

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 14

Not yet answered

Points out of 6

Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000. Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 11$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 15

Not yet answered

Points out of 6

Jesse cuts a circular disk of radius 12, along 2 radii to form 2 sectors, one with a central angle of 120. He makes two circular cones using each sector to form the lateral surface of each cone. What is the ratio of the volume of the smaller cone to the larger cone?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{4} \qquad \textbf{(C)}\ \frac{\sqrt{10}}{10} \qquad \textbf{(D)}\ \frac{\sqrt{5}}{6} \qquad \textbf{(E)}\ \frac{\sqrt{5}}{5}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 16

Not yet answered

Points out of 6

Amy, Beth, and Jo listen to four different songs and discuss which ones they like. No song is liked by all three. Furthermore, for each of the three pairs of the girls, there is at least one song liked by those two girls but disliked by the third. In how many different ways is this possible?

$\textbf{(A)}\ 108 \qquad \textbf{(B)}\ 132 \qquad \textbf{(C)}\ 671 \qquad \textbf{(D)}\ 846 \qquad \textbf{(E)}\ 1105$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 17

Not yet answered

Points out of 6

Square $PQRS$ lies in the first quadrant. Points $(3,0)$, $(5,0)$, $(7,0)$, and $(13,0)$ lie on lines SP, RQ, PQ, and SR, respectively. What is the sum of the coordinates of the center of the square $PQRS$?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 6.2 \qquad \textbf{(C)}\ 6.4 \qquad \textbf{(D)}\ 6.6 \qquad \textbf{(E)}\ 6.8$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 18

Not yet answered

Points out of 6

Let $(a_1, a_2, \dots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \le i \le 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

$\textbf{(A)}\ 120 \qquad \textbf{(B)}\ 512 \qquad \textbf{(C)}\ 1024 \qquad \textbf{(D)}\ 181,440 \qquad \textbf{(E)}\ 362,880$

Select one:

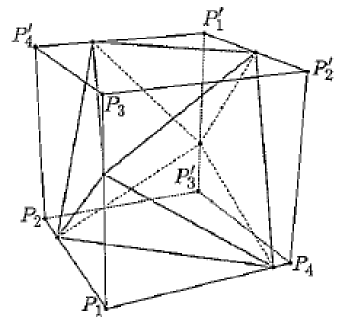
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 19

Not yet answered

Points out of 6

A unit cube has vertices $P_1, P_2, P_3, P_4, P_1', P_2', P_3'$, and P_4'. Vertices P_2, P_3, and P_4 are adjacent to P_1, and for $1 \le i \le 4$, vertices P_i and P_i' are opposite to each other. A regular octahedron has one vertex in each of the segments P_1P_2, P_1P_3, P_1P_4, $P_1'P_2'$, $P_1'P_3'$, and $P_1'P_4'$.



What is the octahedron's side length?

(A) $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{7\sqrt{6}}{16}$ **(C)** $\frac{\sqrt{5}}{2}$ **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\frac{\sqrt{6}}{2}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 20

Not yet answered

Points out of 6

A trapezoid has side lengths 3, 5, 7, and 11. The sums of all the possible areas of the trapezoid can be written in the form of $r_1\sqrt{n_1} + r_2\sqrt{n_2} + r_3$, where r_1, r_2, and r_3 are rational numbers and n_1 and n_2 are positive integers not divisible by the square of any prime. What is the greatest integer less than or equal to $r_1 + r_2 + r_3 + n_1 + n_2$?

(A) 57 **(B)** 59 **(C)** 61 **(D)** 63 **(E)** 65

Select one:

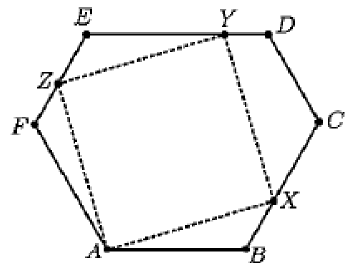
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 21

Not yet answered

Points out of 6

Square $AXYZ$ is inscribed in equiangular hexagon $ABCDEF$ with X on $\overline{BC}$, Y on $\overline{DE}$, and Z on $\overline{EF}$. Suppose that $AB = 40$, and $EF = 41(\sqrt{3} - 1)$.



What is the side-length of the square?

$\textbf{(A)}\ 29\sqrt{3}$ $\textbf{(B)}\ \frac{21}{2}\sqrt{2} + \frac{41}{2}\sqrt{3}$ $\textbf{(C)}\ 20\sqrt{3} + 16$ $\textbf{(D)}\ 20\sqrt{2} + 13\sqrt{3}$ $\textbf{(E)}\ 21\sqrt{6}$

Select one:

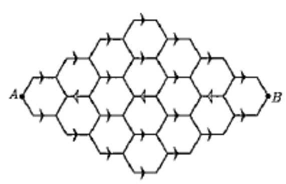
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

A bug travels from A to B along the segments in the hexagonal lattice pictured below. The segments marked with an arrow can be traveled only in the direction of the arrow, and the bug never travels the same segment more than once.



How many different paths are there?

$\textbf{(A)}\ 2112$ $\textbf{(B)}\ 2304$ $\textbf{(C)}\ 2368$ $\textbf{(D)}\ 2384$ $\textbf{(E)}\ 2400$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

Consider all polynomials of a complex variable, $P(z) = 4z^4 + az^3 + bz^2 + cz + d$, where a, b, c, and d are integers, $0 \le d \le c \le b \le a \le 4$, and the polynomial has a zero z_0 with $|z_0| = 1$. What is the sum of all values $P(1)$ over all the polynomials with these properties?

$\textbf{(A)}\ 84 \qquad \textbf{(B)}\ 92 \qquad \textbf{(C)}\ 100 \qquad \textbf{(D)}\ 108 \qquad \textbf{(E)}\ 120$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

Define the function f_1 on the positive integers by setting $f_1(1) = 1$ and if $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ is the prime factorization of $n > 1$, then

$$f_1(n) = (p_1 + 1)^{e_1 - 1}(p_2 + 1)^{e_2 - 1} \cdots (p_k + 1)^{e_k - 1}.$$

For every $m \ge 2$, let $f_m(n) = f_1(f_{m-1}(n))$. For how many N in the range $1 \le N \le 400$ is the sequence $(f_1(N), f_2(N), f_3(N), \dots)$ unbounded?

Note: A sequence of positive numbers is unbounded if for every integer B, there is a member of the sequence greater than B.

$\textbf{(A)}\ 15 \qquad \textbf{(B)}\ 16 \qquad \textbf{(C)}\ 17 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 19$

Select one:

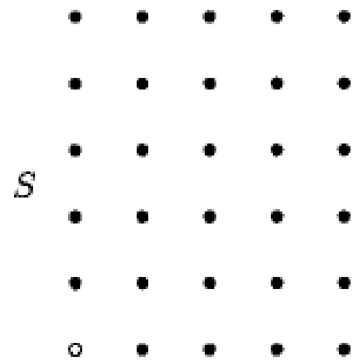
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **25**

Not yet answered

Points out of 6

Let $S = \{(x, y) : x \in \{0, 1, 2, 3, 4\}, y \in \{0, 1, 2, 3, 4, 5\}, \text{ and } (x, y) \neq (0, 0)\}$. Let T be the set of all right triangles whose vertices are in S. For every right triangle $t = \triangle ABC$ with vertices A, B, and C in counter-clockwise order and right angle at A, let $f(t) = \tan(\angle CBA)$.



What is

$$\prod_{t \in T} f(t)?$$

(A) 1 **(B)** $\frac{625}{144}$ **(C)** $\frac{125}{24}$ **(D)** 6 **(E)** $\frac{625}{24}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**