2017 AMC 12B

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Question **1**

Not yet answered

Points out of 6

Kymbrea's comic book collection currently has 30 comic books in it, and she is adding to her collection at the rate of 2 comic books per month. LaShawn's collection currently has 10 comic books in it, and he is adding to his collection at the rate of 6 comic books per month. After how many months will LaShawn's collection have twice as many comic books as Kymbrea's?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **2**

Not yet answered

Points out of 6

Real numbers x, y, and z satisfy the inequalities $0 < x < 1$, $-1 < y < 0$, and $1 < z < 2$. Which of the following numbers is necessarily positive?

$\textbf{(A)}\ y+x^2 \qquad \textbf{(B)}\ y+xz \qquad \textbf{(C)}\ y+y^2 \qquad \textbf{(D)}\ y+2y^2 \qquad \textbf{(E)}\ y+z$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

Supposed that x and y are nonzero real numbers such that $\frac{3x+y}{x-3y} = -2$. What is the value of $\frac{x+3y}{3x-y}$?

$\textbf{(A)}\ -3 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 3$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

Samia set off on her bicycle to visit her friend, traveling at an average speed of 17 kilometers per hour. When she had gone half the distance to her friend's house, a tire went flat, and she walked the rest of the way at 5 kilometers per hour. In all it took her 44 minutes to reach her friend's house. In kilometers rounded to the nearest tenth, how far did Samia walk?

$\textbf{(A)}\ 2.0 \qquad \textbf{(B)}\ 2.2 \qquad \textbf{(C)}\ 2.8 \qquad \textbf{(D)}\ 3.4 \qquad \textbf{(E)}\ 4.4$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

The data set $[6, 19, 33, 33, 39, 41, 41, 43, 51, 57]$ has median $Q_2 = 40$, first quartile $Q_1 = 33$, and third quartile $Q_3 = 43$. An outlier in a data set is a value that is more than 1.5 times the interquartile range below the first quartle (Q_1) or more than 1.5 times the interquartile range above the third quartile (Q_3), where the interquartile range is defined as $Q_3 - Q_1$. How many outliers does this data set have?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

The circle having $(0, 0)$ and $(8, 6)$ as the endpoints of a diameter intersects the x-axis at a second point. What is the x-coordinate of this point?

(A) $4\sqrt{2}$ **(B)** 6 **(C)** $5\sqrt{2}$ **(D)** 8 **(E)** $6\sqrt{2}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

The functions $\sin(x)$ and $\cos(x)$ are periodic with least period 2π. What is the least period of the function $\cos(\sin(x))$?

(A) $\frac{\pi}{2}$ **(B)** π **(C)** 2π **(D)** 4π **(E)** The function is not periodic.

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

The ratio of the short side of a certain rectangle to the long side is equal to the ratio of the long side to the diagonal. What is the square of the ratio of the short side to the long side of this rectangle?

$\textbf{(A)}\ \frac{\sqrt{3}-1}{2} \qquad \textbf{(B)}\ \frac{1}{2} \qquad \textbf{(C)}\ \frac{\sqrt{5}-1}{2} \qquad \textbf{(D)}\ \frac{\sqrt{2}}{2} \qquad \textbf{(E)}\ \frac{\sqrt{6}-1}{2}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

A circle has center $(-10,-4)$ and has radius 13. Another circle has center $(3,9)$ and radius $\sqrt{65}$. The line passing through the two points of intersection of the two circles has equation $x+y=c$. What is c?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 3\sqrt{3} \qquad \textbf{(C)}\ 4\sqrt{2} \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ \frac{13}{2}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **10**

Not yet answered

Points out of 6

At Typico High School, 60% of the students like dancing, and the rest dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 90% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

$\textbf{(A)}\ 10\% \qquad \textbf{(B)}\ 12\% \qquad \textbf{(C)}\ 20\% \qquad \textbf{(D)}\ 25\% \qquad \textbf{(E)}\ \frac{100}{3}\%$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **11**

Not yet answered

Points out of 6

Call a positive integer *monotonous* if it is a one-digit number or its digits, when read from left to right, form either a strictly increasing or a strictly decreasing sequence. For example, 3, 23578, and 987620 are monotonous, but 88, 7434, and 23557 are not. How many monotonous positive integers are there?

$\textbf{(A)}\ 1024 \qquad \textbf{(B)}\ 1524 \qquad \textbf{(C)}\ 1533 \qquad \textbf{(D)}\ 1536 \qquad \textbf{(E)}\ 2048$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **12**

Not yet answered

Points out of 6

What is the sum of the roots of $z^{12} = 64$ that have a positive real part?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ \sqrt{2}+2\sqrt{3} \qquad \textbf{(D)}\ 2\sqrt{2}+\sqrt{6} \qquad \textbf{(E)}\ (1+\sqrt{3})+(1+\sqrt{3})i$

Select one:

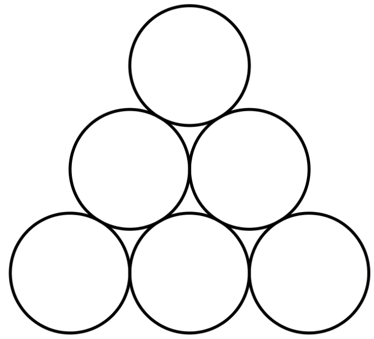
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **13**

Not yet answered

Points out of 6

In the figure below, 3 of the 6 disks are to be painted blue, 2 are to be painted red, and 1 is to be painted green. Two paintings that can be obtained from one another by a rotation or a reflection of the entire figure are considered the same.



How many different paintings are possible?

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 15$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **14**

Not yet answered

Points out of 6

An ice-cream novelty item consists of a cup in the shape of a 4-inch-tall frustum of a right circular cone, with a 2-inch-diameter base at the bottom and a 4-inch-diameter base at the top, packed solid with ice cream, together with a solid cone of ice cream of height 4 inches, whose base, at the bottom, is the top base of the frustum. What is the total volume of the ice cream, in cubic inches?

$\textbf{(A)}\ 8\pi \qquad \textbf{(B)}\ \frac{28\pi}{3} \qquad \textbf{(C)}\ 12\pi \qquad \textbf{(D)}\ 14\pi \qquad \textbf{(E)}\ \frac{44\pi}{3}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **15**

Not yet answered

Points out of 6

Let ABC be an equilateral triangle. Extend side $\overline{AB}$ beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side $\overline{BC}$ beyond C to a point C' so that $CC' = 3BC$, and extend side $\overline{CA}$ beyond A to a point A' so that $AA' = 3CA$. What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

$\textbf{(A)}\ 9:1 \qquad \textbf{(B)}\ 16:1 \qquad \textbf{(C)}\ 25:1 \qquad \textbf{(D)}\ 36:1 \qquad \textbf{(E)}\ 37:1$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **16**

Not yet answered

Points out of 6

The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

$\textbf{(A)}\ \frac{1}{21} \qquad \textbf{(B)}\ \frac{1}{19} \qquad \textbf{(C)}\ \frac{1}{18} \qquad \textbf{(D)}\ \frac{1}{2} \qquad \textbf{(E)}\ \frac{11}{21}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **17**

Not yet answered

Points out of 6

A coin is biased in such a way that on each toss the probability of heads is $\frac{2}{3}$ and the probability of tails is $\frac{1}{3}$. The outcomes of the tosses are independent. A player has the choice of playing Game A or Game B. In Game A she tosses the coin three times and wins if all three outcomes are the same. In Game B she tosses the coin four times and wins if both the outcomes of the first and second tosses are the same and the outcomes of the third and fourth tosses are the same. How do the chances of winning Game A compare to the chances of winning Game B?

$\textbf{(A)}$ The probability of winning Game A is $\frac{4}{81}$ less than the probability of winning Game B.

$\textbf{(B)}$ The probability of winning Game A is $\frac{2}{81}$ less than the probability of winning Game B.

$\textbf{(C)}$ The probabilities are the same.

$\textbf{(D)}$ The probability of winning Game A is $\frac{2}{81}$ greater than the probability of winning Game B.

$\textbf{(E)}$ The probability of winning Game A is $\frac{4}{81}$ greater than the probability of winning Game B.

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **18**

Not yet answered

Points out of 6

The diameter AB of a circle of radius 2 is extended to a point D outside the circle so that $BD = 3$. Point E is chosen so that $ED = 5$ and line ED is perpendicular to line AD. Segment AE intersects the circle at a point C between A and E. What is the area of $\triangle ABC$?

$\textbf{(A)}\ \frac{120}{37}$ $\quad\textbf{(B)}\ \frac{140}{39}$ $\quad\textbf{(C)}\ \frac{145}{39}$ $\quad\textbf{(D)}\ \frac{140}{37}$ $\quad\textbf{(E)}\ \frac{120}{31}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **19**

Not yet answered

Points out of 6

Let $N = 123456789101112\dots4344$ be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 18 \qquad \textbf{(E)}\ 44$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **20**

Not yet answered

Points out of 6

Real numbers x and y are chosen independently and uniformly at random from the interval $(0, 1)$. What is the probability that $\lfloor \log_2 x \rfloor = \lfloor \log_2 y \rfloor$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r ?

$\textbf{(A)}\ \frac{1}{8} \qquad \textbf{(B)}\ \frac{1}{6} \qquad \textbf{(C)}\ \frac{1}{4} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{2}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **21**

Not yet answered

Points out of 6

Last year, Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

$\textbf{(A)}\ 92 \qquad \textbf{(B)}\ 94 \qquad \textbf{(C)}\ 96 \qquad \textbf{(D)}\ 98 \qquad \textbf{(E)}\ 100$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

Abby, Bernardo, Carl, and Debra play a game in which each of them starts with four coins. The game consists of four rounds. In each round, four balls are placed in an urn---one green, one red, and two white. The players each draw a ball at random without replacement. Whoever gets the green ball gives one coin to whoever gets the red ball. What is the probability that, at the end of the fourth round, each of the players has four coins?

(A) $\frac{7}{576}$ **(B)** $\frac{5}{192}$ **(C)** $\frac{1}{36}$ **(D)** $\frac{5}{144}$ **(E)** $\frac{7}{48}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

The graph of $y = f(x)$, where $f(x)$ is a polynomial of degree 3, contains points $A(2,4)$, $B(3,9)$, and $C(4,16)$. Lines AB, AC, and BC intersect the graph again at points D, E, and F, respectively, and the sum of the x-coordinates of D, E, and F is 24. What is $f(0)$?

(A) -2 **(B)** 0 **(C)** 2 **(D)** $\frac{24}{5}$ **(E)** 8

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **24**

Not yet answered

Points out of 6

Quadrilateral $ABCD$ has right angles at B and C, $\triangle ABC \sim \triangle BCD$, and $AB > BC$. There is a point E in the interior of $ABCD$ such that $\triangle ABC \sim \triangle CEB$ and the area of $\triangle AED$ is 17 times the area of $\triangle CEB$. What is $\dfrac{AB}{BC}$?

$\textbf{(A)}\ 1+\sqrt{2} \qquad \textbf{(B)}\ 2+\sqrt{2} \qquad \textbf{(C)}\ \sqrt{17} \qquad \textbf{(D)}\ 2+\sqrt{5} \qquad \textbf{(E)}\ 1+2\sqrt{3}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **25**

Not yet answered

Points out of 6

A set of n people participate in an online video basketball tournament. Each person may be a member of any number of 5-player teams, but no two teams may have exactly the same 5 members. The site statistics show a curious fact: The average, over all subsets of size 9 of the set of n participants, of the number of complete teams whose members are among those 9 people is equal to the reciprocal of the average, over all subsets of size 8 of the set of n participants, of the number of complete teams whose members are among those 8 people. How many values n, $9 \leq n \leq 2017$, can be the number of participants?

$\textbf{(A)}\ 477 \qquad \textbf{(B)}\ 482 \qquad \textbf{(C)}\ 487 \qquad \textbf{(D)}\ 557 \qquad \textbf{(E)}\ 562$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**