2019 AMC 12B

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Question 1

Not yet answered

Points out of 6

Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the smaller container to the volume of the larger container?

(A) $\frac{5}{8}$

(B) $\frac{4}{5}$

(C) $\frac{7}{8}$

(D) $\frac{9}{10}$

(E) $\frac{11}{12}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

(A) 11

(B) 15

(C) 19

(D) 21

(E) 27

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2,1)$ is $A'(2,-1)$ and the image of $B(-1,4)$ is $B'(1,-4)$?

(A) reflection in the y-axis

(B) counterclockwise rotation around the origin by 90°

(C) translation by 3 units to the right and 5 units down

(D) reflection in the x-axis

(E) clockwise rotation about the origin by180°

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

A positive integer n satisfies the equation $(n+1)!+(n+2)!=440\cdot n!$. What is the sum of the digits of n?

(A) 2

(B) 5

(C) 10

(D) 12

(E) 15

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n?

$\textbf{(A)}\ 18$
$\textbf{(B)}\ 21$
$\textbf{(C)}\ 24$
$\textbf{(D)}\ 25$
$\textbf{(E)}\ 28$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

$\textbf{(A)}\ 0$
$\textbf{(B)}\ 2$
$\textbf{(C)}\ 4$
$\textbf{(D)}\ 8$
$\textbf{(E)}\ \text{infinitely many}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

(A) -5
(B) 0
(C) 5
(D) $\frac{15}{4}$
(E) $\frac{35}{4}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

Let $f(x) = x^2(1-x)^2$. What is the value of the sum

$$f\left(\frac{1}{2019}\right) - f\left(\frac{2}{2019}\right) + f\left(\frac{3}{2019}\right) - f\left(\frac{4}{2019}\right) + \cdots + f\left(\frac{2017}{2019}\right) - f\left(\frac{2018}{2019}\right)?$$

(A) 0
(B) $\frac{1}{2019^4}$
(C) $\frac{2018^2}{2019^4}$
(D) $\frac{2020^2}{2019^4}$
(E) 1

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

For how many integral values of x can a triangle of positive area be formed having side lengths $\log_2 x$, $\log_4 x$, and 3?

$\textbf{(A)}\ 57$

$\textbf{(B)}\ 59$

$\textbf{(C)}\ 61$

$\textbf{(D)}\ 62$

$\textbf{(E)}\ 63$

Select one:

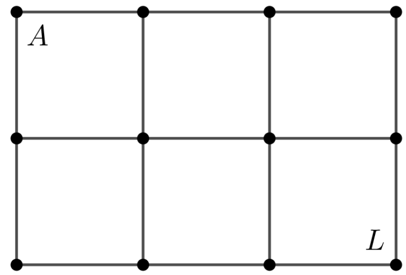
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 10

Not yet answered

Points out of 6

The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L, without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.)



How many different routes can Paula take?

$\textbf{(A)}\ 0$
$\textbf{(B)}\ 1$
$\textbf{(C)}\ 2$
$\textbf{(D)}\ 3$
$\textbf{(E)}\ 4$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 11

Not yet answered

Points out of 6

How many unordered pairs of edges of a given cube determine a plane?

$\textbf{(A)}\ 21$
$\textbf{(B)}\ 28$
$\textbf{(C)}\ 36$
$\textbf{(D)}\ 42$
$\textbf{(E)}\ 66$

Select one:

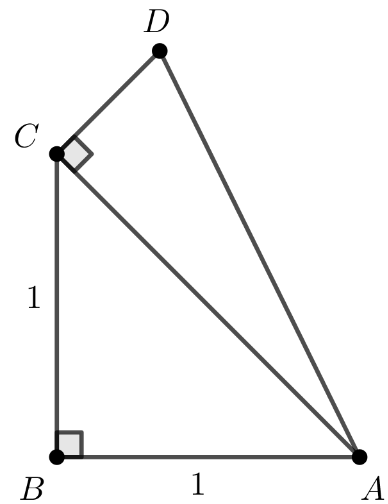
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 12

Not yet answered

Points out of 6

Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?



(A) $\frac{1}{3}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{3}{4}$

(D) $\frac{7}{9}$

(E) $\frac{\sqrt{3}}{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question **13**

Not yet answered

Points out of 6

A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

$\textbf{(A)}\ \frac{1}{4}$

$\textbf{(B)}\ \frac{2}{7}$

$\textbf{(C)}\ \frac{1}{3}$

$\textbf{(D)}\ \frac{3}{8}$

$\textbf{(E)}\ \frac{3}{7}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **14**

Not yet answered

Points out of 6

Let S be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of S?

$\textbf{(A)}\ 98$

$\textbf{(B)}\ 100$

$\textbf{(C)}\ 117$

$\textbf{(D)}\ 119$

$\textbf{(E)}\ 121$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **15**

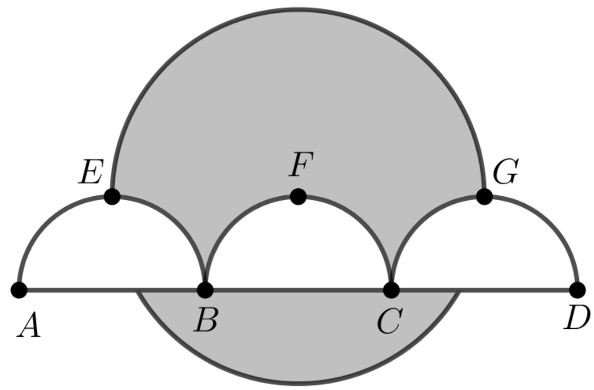
Not yet answered

Points out of 6

As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$ .Three semicircles of radius 1, $\widehat{AEB}$, $\widehat{BFC}$, and $\widehat{CGD}$, have their diameters on $\overline{AD}$, lie in the same halfplane determined by line AD, and are tangent to line EG at E ,F, and G, respectively. A circle of radius 2 has its center at F. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where a, b, c, and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?



$\textbf{(A)}$ 13
$\textbf{(B)}$ 14
$\textbf{(C)}$ 15
$\textbf{(D)}$ 16
$\textbf{(E)}$ 17

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 16

Not yet answered

Points out of 6

Lily pads numbered from 0 to 11 lie in a row on a pond. Fiona the frog sits on pad 0, a morsel of food sits on pad 10, and predators sit on pads 3 and 6. At each unit of time the frog jumps either to the next higher numbered pad or to the pad after that, each with probability $\frac{1}{2}$, independently from previous jumps. What is the probability that Fiona skips over pads 3 and 6 and lands on pad 10?

$\textbf{(A)}\ \frac{15}{256}$

$\textbf{(B)}\ \frac{1}{16}$

$\textbf{(C)}\ \frac{15}{128}$

$\textbf{(D)}\ \frac{1}{8}$

$\textbf{(E)}\ \frac{1}{4}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 17

Not yet answered

Points out of 6

How many nonzero complex numbers z have the property that $0, z,$ and z^3, when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

$\textbf{(A)}\ 0$

$\textbf{(B)}\ 1$

$\textbf{(C)}\ 2$

$\textbf{(D)}\ 4$

$\textbf{(E)}\ \text{infinitely many}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 18

Not yet answered

Points out of 6

Square pyramid $ABCDE$ has base $ABCD$, which measures 3 cm on a side, and altitude $\overline{AE}$ perpendicular to the base, which measures 6 cm. Point P lies on $\overline{BE}$, one third of the way from B to E; point Q lies on $\overline{DE}$, one third of the way from D to E; and point R lies on $\overline{CE}$, two thirds of the way from C to E. What is the area, in square centimeters, of $\triangle PQR$?

$\textbf{(A)}\ \frac{3\sqrt{2}}{2}$

$\textbf{(B)}\ \frac{3\sqrt{3}}{2}$

$\textbf{(C)}\ 2\sqrt{2}$

$\textbf{(D)}\ 2\sqrt{3}$

$\textbf{(E)}\ 3\sqrt{2}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question **19**

Not yet answered

Points out of 6

Raashan, Sylvia, and Ted play the following game. Each starts with $\$1$. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives $\$1$ to that player. What is the probability that after the bell has rung 2019 times, each player will have $\$1$? (For example, Raashan and Ted may each decide to give $\$1$ to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have $\$0$, Sylvia would have $\$2$, and Ted would have $\$1$, and and that is the end of the first round of play. In the second round Raashan has no money to give, but Sylvia and Ted might choose each other to give their $\$1$ to, and the holdings will be the same at the end of the second round.)

$\textbf{(A)}\ \frac{1}{7}$

$\textbf{(B)}\ \frac{1}{4}$

$\textbf{(C)}\ \frac{1}{3}$

$\textbf{(D)}\ \frac{1}{2}$

$\textbf{(E)}\ \frac{2}{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 20

Not yet answered

Points out of 6

Points $A(6, 13)$ and $B(12, 11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

(A) $\frac{83\pi}{8}$
(B) $\frac{21\pi}{2}$
(C) $\frac{85\pi}{8}$
(D) $\frac{43\pi}{4}$
(E) $\frac{87\pi}{8}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 21

Not yet answered

Points out of 6

How many quadratic polynomials with real coefficients are there such that the set of roots equals the set of coefficients? (For clarification: If the polynomial is $ax^2 + bx + c, a \neq 0,$ and the roots are r and s, then the requirement is that $\{a, b, c\} = \{r, s\}$.)

(A) 3
(B) 4
(C) 5
(D) 6
(E) infinitely many

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

(A) $[9, 26]$
(B) $[27, 80]$
(C) $[81, 242]$
(D) $[243, 728]$
(E) $[729, \infty)$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55
(B) 60
(C) 65
(D) 70
(E) 75

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$. Let S denote all points in the complex plane of the form $a + b\omega + c\omega^2$, where $0 \le a \le 1, 0 \le b \le 1$, and $0 \le c \le 1$. What is the area of S?

(A) $\frac{1}{2}\sqrt{3}$
(B) $\frac{3}{4}\sqrt{3}$
(C) $\frac{3}{2}\sqrt{3}$
(D) $\frac{1}{2}\pi\sqrt{3}$
(E) π

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 25

Not yet answered

Points out of 6

Let $ABCD$ be a convex quadrilateral with $BC = 2$ and $CD = 6$. Suppose that the centroids of $\triangle ABC$, $\triangle BCD$, and $\triangle ACD$ form the vertices of an equilateral triangle. What is the maximum possible value of $ABCD$?

(A) 27
(B) $16\sqrt{3}$
(C) $12 + 10\sqrt{3}$
(D) $9 + 12\sqrt{3}$
(E) 30

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**