2022 AMC 12B

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Question 1

Not yet answered

Points out of 6

Define $x\diamondsuit y$ to be $|x-y|$ for all real numbers x and y. What is the value of

$$(1\diamondsuit(2\diamondsuit 3))-((1\diamondsuit 2)\diamondsuit 3)?$$

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ 2$

Select one:

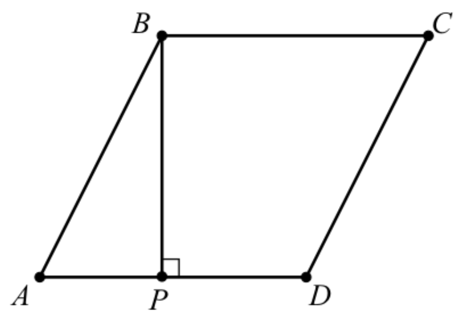
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

In rhombus $ABCD$, point P lies on segment $\overline{AD}$ so that $\overline{BP} \perp \overline{AD}$, $AP=3$, and $PD=2$. What is the area of $ABCD$? (Note: The figure is not drawn to scale.)



$\textbf{(A)}\ 3\sqrt{5} \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 6\sqrt{5} \qquad \textbf{(D)}\ 20 \qquad \textbf{(E)}\ 25$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

How many of the first ten numbers of the sequence $121, 11211, 1112111, \ldots$ are prime numbers?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ 2 \qquad \textbf{(D)}\ 3 \qquad \textbf{(E)}\ 4$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

For how many values of the constant k will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

(A) 6 **(B)** 8 **(C)** 9 **(D)** 14 **(E)** 16

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

The point $(-1, -2)$ is rotated 270° counterclockwise about the point $(3, 1)$. What are the coordinates of its new position?

(A) $(-3, -4)$ **(B)** $(0, 5)$ **(C)** $(2, -1)$ **(D)** $(4, 3)$ **(E)** $(6, -3)$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

Consider the following 100 sets of 10 elements each:

$$\begin{array}{l}\{1, 2, 3, \ldots, 10\},\\ \{11, 12, 13, \ldots, 20\},\\ \{21, 22, 23, \ldots, 30\},\\ \vdots\\ \{991, 992, 993, \ldots, 1000\}.\end{array}$$

How many of these sets contain exactly two multiples of 7?

(A) 40 **(B)** 42 **(C)** 43 **(D)** 49 **(E)** 50

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 9 \qquad \textbf{(D)}\ 11 \qquad \textbf{(E)}\ 13$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

What is the graph of $y^4 + 1 = x^4 + 2y^2$ in the coordinate plane?

$\textbf{(A)}$ two intersecting parabolas $\qquad \textbf{(B)}$ two nonintersecting parabolas $\qquad \textbf{(C)}$ two intersecting circles

$\textbf{(D)}$ a circle and a hyperbola $\qquad \textbf{(E)}$ a circle and two parabolas

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

The sequence $a_0, a_1, a_2, \cdots$ is a strictly increasing arithmetic sequence of positive integers such that

$$2^{a_7} = 2^{27} \cdot a_7.$$

What is the minimum possible value of a_2?

$\textbf{(A)}\ 8 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 22$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **10**

Not yet answered

Points out of 6

Regular hexagon $ABCDEF$ has side length 2. Let G be the midpoint of $\overline{AB}$, and let H be the midpoint of $\overline{DE}$. What is the perimeter of $GCHF$?

$\textbf{(A)}\ 4\sqrt{3} \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 4\sqrt{5} \qquad \textbf{(D)}\ 4\sqrt{7} \qquad \textbf{(E)}\ 12$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **11**

Not yet answered

Points out of 6

Let $f(n) = \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$, where $i = \sqrt{-1}$. What is $f(2022)$?

$\textbf{(A)}\ -2 \qquad \textbf{(B)}\ -1 \qquad \textbf{(C)}\ 0 \qquad \textbf{(D)}\ \sqrt{3} \qquad \textbf{(E)}\ 2$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **12**

Not yet answered

Points out of 6

Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2?

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{19}{27} \qquad \textbf{(C)}\ \frac{59}{81} \qquad \textbf{(D)}\ \frac{61}{81} \qquad \textbf{(E)}\ \frac{7}{9}$

Select one:

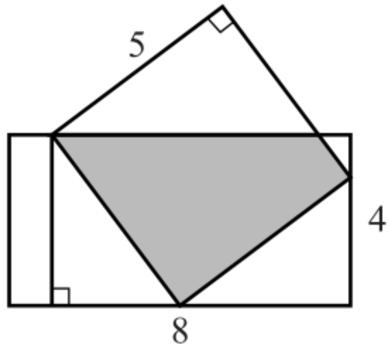
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 13

Not yet answered

Points out of 6

The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?



(A) $15\frac{1}{8}$ **(B)** $15\frac{3}{8}$ **(C)** $15\frac{1}{2}$ **(D)** $15\frac{5}{8}$ **(E)** $15\frac{7}{8}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 14

Not yet answered

Points out of 6

The graph of $y = x^2 + 2x - 15$ intersects the x-axis at points A and C and the y-axis at point B. What is $\tan(\angle ABC)$?

(A) $\dfrac{1}{7}$ **(B)** $\dfrac{1}{4}$ **(C)** $\dfrac{3}{7}$ **(D)** $\dfrac{1}{2}$ **(E)** $\dfrac{4}{7}$

Select one:

- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question **15**

Not yet answered

Points out of 6

One of the following numbers is not divisible by any prime number less than 10. Which is it?

$\textbf{(A)}\ 2^{606}-1 \qquad \textbf{(B)}\ 2^{606}+1 \qquad \textbf{(C)}\ 2^{607}-1$
$\textbf{(D)}\ 2^{607}+1 \qquad \text{(E)}\ 2^{607}+3^{607}$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question **16**

Not yet answered

Points out of 6

Suppose x and y are positive real numbers such that

$$x^y = 2^{64} \text{ and } (\log_2 x)^{\log_2 y} = 2^7.$$

What is the greatest possible value of $\log_2 y$?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 3+\sqrt{2} \qquad \textbf{(D)}\ 4+\sqrt{3} \qquad \textbf{(E)}\ 7$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question **17**

Not yet answered

Points out of 6

How many 4×4 arrays whose entries are 0s and 1s are there such that the row sums (the sum of the entries in each row) are 1, 2, 3, and 4, in some order, and the column sums (the sum of the entries in each column) are also 1, 2, 3, and 4, in some order? For example, the array

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

satisfies the condition.

$\textbf{(A)}\ 144 \qquad \textbf{(B)}\ 240 \qquad \textbf{(C)}\ 336 \qquad \textbf{(D)}\ 576 \qquad \textbf{(E)}\ 624$

Select one:

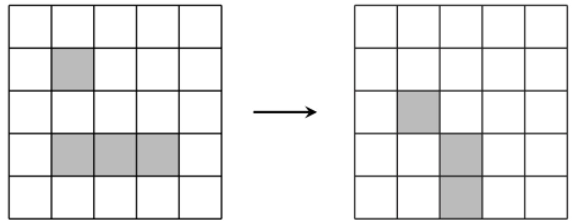
- A
- B
- C
- D
- E
- Leave blank (1.5 points)

Question **18**

Not yet answered

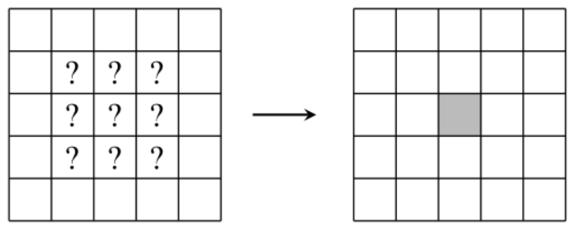
Points out of 6

Each square in a 5×5 grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules: Any filled square with two or three filled neighbors remains filled. Any empty square with exactly three filled neighbors becomes a filled square. All other squares remain empty or become empty. A sample transformation is shown in the figure below.



Initial Transformed

Suppose the 5×5 grid has a border of empty squares surrounding a 3×3 subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)



Initial Transformed

$\textbf{(A)}\ 14 \qquad \textbf{(B)}\ 18 \qquad \textbf{(C)}\ 22 \qquad \textbf{(D)}\ 26 \qquad \textbf{(E)}\ 30$

Select one:

- ○ **A**
- ○ **B**
- ○ **C**
- ○ **D**
- ○ **E**
- ○ **Leave blank (1.5 points)**

Question **19**

Not yet answered

Points out of 6

In $\triangle ABC$ medians $\overline{\text{AD}}$ and $\overline{\text{BE}}$ intersect at G and $\triangle AGE$ is equilateral. Then $\cos(C)$ can be written as $\frac{m\sqrt{p}}{n}$, where m and n are relatively prime positive integers and p is a positive integer not divisible by the square of any prime. What is $m+n+p$?

$\textbf{(A)}\ 44 \qquad \textbf{(B)}\ 48 \qquad \textbf{(C)}\ 52 \qquad \textbf{(D)}\ 56 \qquad \textbf{(E)}\ 60$

Select one:

- ○ **A**
- ○ **B**
- ○ **C**
- ○ **D**
- ○ **E**
- ○ **Leave blank (1.5 points)**

Question **20**

Not yet answered

Points out of 6

Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial $x^2 + x + 1$, the remainder is $x + 2$, and when $P(x)$ is divided by the polynomial $x^2 + 1$, the remainder is $2x + 1$. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

(A) 10 **(B)** 13 **(C)** 19 **(D)** 20 **(E)** 23

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **21**

Not yet answered

Points out of 6

Let S be the set of circles in the coordinate plane that are tangent to each of the three circles with equations $x^2 + y^2 = 4$, $x^2 + y^2 = 64$, and $(x - 5)^2 + y^2 = 3$. What is the sum of the areas of all circles in S?

(A) 48π **(B)** 68π **(C)** 96π **(D)** 102π **(E)** 136π

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question **22**

Not yet answered

Points out of 6

Ant Amelia starts on the number line at 0 and crawls in the following manner. For $n = 1, 2, 3,$ Amelia chooses a time duration t_n and an increment x_n independently and uniformly at random from the interval $(0, 1)$. During the nth step of the process, Amelia moves x_n units in the positive direction, using up t_n minutes. If the total elapsed time has exceeded 1 minute during the nth step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?

(A) $\frac{1}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{5}{6}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

Let $x_0, x_1, x_2, \ldots$ be a sequence of numbers, where each x_k is either 0 or 1. For each positive integer n, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k$$

Suppose $7S_n \equiv 1 \pmod{2^n}$ for all $n \geq 1$. What is the value of the sum

$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}$$

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

Select one:

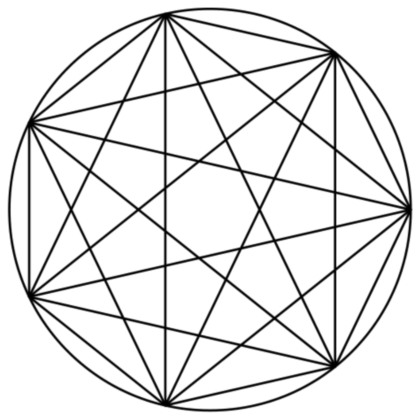
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

The figure below depicts a regular 7-gon inscribed in a unit circle.



What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?

$\textbf{(A)}\ 49 \qquad \textbf{(B)}\ 98 \qquad \textbf{(C)}\ 147 \qquad \textbf{(D)}\ 168 \qquad \textbf{(E)}\ 196$

Select one:

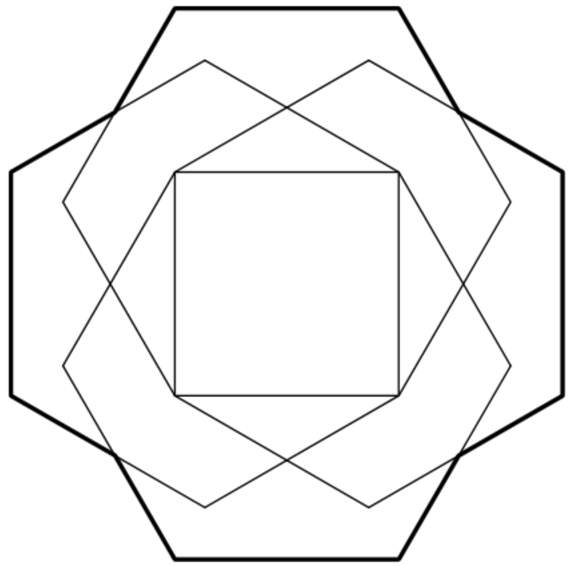
- A
- B
- C
- D
- E
- **Leave blank (1.5 points)**

Question 25

Not yet answered

Points out of 6

Four regular hexagons surround a square with side length 1, each one sharing an edge with the square, as shown in the figure below. The area of the resulting 12-sided outer nonconvex polygon can be written as $m\sqrt{n}+p$, where m, n, and p are integers and n is not divisible by the square of any prime. What is $m+n+p$?



$\textbf{(A)}\ -12 \qquad \textbf{(B)}\ -4 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 32$

Select one:

- A
- B
- C
- D
- E
- Leave blank (1.5 points)