2023 AMC 12B

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Question 1

Not yet answered

Points out of 6

Mrs. Jones is pouring orange juice into four identical glasses for her four sons. She fills the first three glasses completely but runs out of juice when the fourth glass is only $\frac{1}{3}$ full. What fraction of a glass must Mrs. Jones pour from each of the first three glasses into the fourth glass so that all four glasses will have the same amount of juice?

$\textbf{(A)}\ \frac{1}{12}$ $\textbf{(B)}\ \frac{1}{4}$ $\textbf{(C)}\ \frac{1}{6}$ $\textbf{(D)}\ \frac{1}{8}$ $\textbf{(E)}\ \frac{2}{9}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 2

Not yet answered

Points out of 6

Carlos went to a sports store to buy running shoes. Running shoes were on sale, with prices reduced by 20% on every pair of shoes. Carlos also knew that he had to pay a 7.5% sales tax on the discounted price. He had $\$43$. What is the original (before discount) price of the most expensive shoes he could afford to buy?

$\textbf{(A)}\ \$46$ $\textbf{(B)}\ \$50$ $\textbf{(C)}\ \$48$ $\textbf{(D)}\ \$47$ $\textbf{(E)}\ \$49$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 3

Not yet answered

Points out of 6

A $3-4-5$ right triangle is inscribed in circle A, and a $5-12-13$ right triangle is inscribed in circle B. What is the ratio of the area of circle A to the area of circle B?

$\textbf{(A)}\ \frac{9}{25} \qquad \textbf{(B)}\ \frac{1}{9} \qquad \textbf{(C)}\ \frac{1}{5} \qquad \textbf{(D)}\ \frac{25}{169} \qquad \textbf{(E)}\ \frac{4}{25}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 4

Not yet answered

Points out of 6

Jackson's paintbrush makes a narrow strip with a width of 6.5 millimeters. Jackson has enough paint to make a strip 25 meters long. How many square centimeters of paper could Jackson cover with paint?

$\textbf{(A)}\ 162,500 \qquad \textbf{(B)}\ 162.5 \qquad \textbf{(C)}\ 1,625 \qquad \textbf{(D)}\ 1,625,000 \qquad \textbf{(E)}\ 16,250$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

You are playing a game. A 2×1 rectangle covers two adjacent squares (oriented either horizontally or vertically) of a 3×3 grid of squares, but you are not told which two squares are covered. Your goal is to find at least one square that is covered by the rectangle. A "turn" consists of you guessing a square, after which you are told whether that square is covered by the hidden rectangle. What is the minimum number of turns you need to ensure that at least one of your guessed squares is covered by the rectangle?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 5 \qquad (\mathrm{C})\ 4 \qquad (\mathrm{D})\ 8 \qquad (\mathrm{E})\ 6$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 6

Not yet answered

Points out of 6

When the roots of the polynomial

$$P(x) = (x-1)^1(x-2)^2(x-3)^3 \cdots (x-10)^{10}$$

are removed from the number line, what remains is the union of 11 disjoint open intervals. On how many of these intervals is $P(x)$ positive?

$\textbf{(A)}\ 3 \qquad \textbf{(B)}\ 7 \qquad (\mathrm{C})\ 6 \qquad (\mathrm{D})\ 4 \qquad (\mathrm{E})\ 5$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 7

Not yet answered

Points out of 6

For how many integers n does the expression

$$\sqrt{\frac{\log(n^2) - (\log n)^2}{\log n - 3}}$$

represent a real number, where $\log$ denotes the base 10 logarithm?

$\textbf{(A)}\ 900 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 902 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ 901$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 8

Not yet answered

Points out of 6

How many nonempty subsets B of $\{0, 1, 2, 3, \cdots, 12\}$ have the property that the number of elements in B is equal to the least element of B? For example, $B = \{4, 6, 8, 11\}$ satisfies the condition.

$\textbf{(A)}\ 256 \qquad \textbf{(B)}\ 136 \qquad \textbf{(C)}\ 108 \qquad \textbf{(D)}\ 144 \qquad \textbf{(E)}\ 156$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 9

Not yet answered

Points out of 6

What is the area of the region in the coordinate plane defined by

$$||x|-1|+||y|-1| \leq 1?$$

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 8 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 12$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 10

Not yet answered

Points out of 6

In the xy-plane, a circle of radius 4 with center on the positive x-axis is tangent to the y-axis at the origin, and a circle with radius 10 with center on the positive y-axis is tangent to the x-axis at the origin. What is the slope of the line passing through the two points at which these circles intersect?

$\textbf{(A)}\ \frac{2}{7} \qquad \textbf{(B)}\ \frac{3}{7} \qquad \textbf{(C)}\ \frac{2}{\sqrt{29}} \qquad \textbf{(D)}\ \frac{1}{\sqrt{29}} \qquad \textbf{(E)}\ \frac{2}{5}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 11

Not yet answered

Points out of 6

What is the maximum area of an isosceles trapezoid that has legs of length 1 and one base twice as long as the other?

$\textbf{(A)}\ \frac{5}{4} \qquad \textbf{(B)}\ \frac{8}{7} \qquad \textbf{(C)}\ \frac{5\sqrt{2}}{4} \qquad \textbf{(D)}\ \frac{3}{2} \qquad \textbf{(E)}\ \frac{3\sqrt{3}}{4}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 12

Not yet answered

Points out of 6

For complex number $u = a + bi$ and $v = c + di$ (where $i = \sqrt{-1}$), define the binary operation

$$u \otimes v = ac + bdi$$

Suppose z is a complex number such that $z \otimes z = z^2 + 40$. What is $|z|$?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 5 \qquad \textbf{(C)}\ \sqrt{5} \qquad \textbf{(D)}\ \sqrt{10} \qquad \textbf{(E)}\ 5\sqrt{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 13

Not yet answered

Points out of 6

A rectangular box $\mathcal{P}$ has distinct edge lengths a, b, and c. The sum of the lengths of all 12 edges of $\mathcal{P}$ is 13, the areas of all 6 faces of $\mathcal{P}$ is $\frac{11}{2}$, and the volume of $\mathcal{P}$ is $\frac{1}{2}$. What is the length of the longest interior diagonal connecting two vertices of $\mathcal{P}$?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ \frac{3}{8} \qquad \textbf{(C)}\ \frac{9}{8} \qquad \textbf{(D)}\ \frac{9}{4} \qquad \textbf{(E)}\ \frac{3}{2}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 14

Not yet answered

Points out of 6

For how many ordered pairs (a, b) of integers does the polynomial $x^3 + ax^2 + bx + 6$ have 3 distinct integer roots?

$\textbf{(A)}\ 5 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 4$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 15

Not yet answered

Points out of 6

Suppose a, b, and c are positive integers such that

$$\frac{a}{14}+\frac{b}{15}=\frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both, then $\gcd(c,210)=1$.

II. If $\gcd(c,210)=1$, then $\gcd(a,14)=1$ or $\gcd(b,15)=1$ or both.

III. $\gcd(c,210)=1$ if and only if $\gcd(a,14)=\gcd(b,15)=1$.

$\textbf{(A)}$ I, II, and III $\quad$ $\textbf{(B)}$ I only $\quad$ $\textbf{(C)}$ I and II only

$\textbf{(D)}$ III only $\quad$ $\textbf{(E)}$ II and III only

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 16

Not yet answered

Points out of 6

In the state of Coinland, coins have values $6,10$, and 15 cents. Suppose x is the value in cents of the most expensive item in Coinland that cannot be purchased using these coins with exact change. What is the sum of the digits of x?

$\textbf{(A)}$ 8 $\quad$ $\textbf{(B)}$ 10 $\quad$ $\textbf{(C)}$ 7 $\quad$ $\textbf{(D)}$ 11 $\quad$ $\textbf{(E)}$ 9

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 17

Not yet answered

Points out of 6

Triangle ABC has side lengths in arithmetic progression, and the smallest side has length 6. If the triangle has an angle of 120°, what is the area of ABC?

$\textbf{(A)}\ 12\sqrt{3} \qquad \textbf{(B)}\ 8\sqrt{6} \qquad \textbf{(C)}\ 14\sqrt{2} \qquad \textbf{(D)}\ 20\sqrt{2} \qquad \textbf{(E)}\ 15\sqrt{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 18

Not yet answered

Points out of 6

Last academic year Yolanda and Zelda took different courses that did not necessarily administer the same number of quizzes during each of the two semesters. Yolanda's average on all the quizzes she took during the first semester was 3 points higher than Zelda's average on all the quizzes she took during the first semester. Yolanda's average on all the quizzes she took during the second semester was 18 points higher than her average for the first semester and was again 3 points higher than Zelda's average on all the quizzes Zelda took during her second semester. Which one of the following statements cannot possibly be true?

$\textbf{(A)}$ Yolanda's quiz average for the academic year was 22 points higher than Zelda's.

$\textbf{(B)}$ Zelda's quiz average for the academic year was higher than Yolanda's.

$\textbf{(C)}$ Yolanda's quiz average for the academic year was 3 points higher than Zelda's.

$\textbf{(D)}$ Zelda's quiz average for the academic year equaled Yolanda's.

$\textbf{(E)}$ If Zelda had scored 3 points higher on each quiz she took, then she would have had the same average for the academic year as Yolanda.

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 19

Not yet answered

Points out of 6

Each of 2023 balls is randomly placed into one of 3 bins. Which of the following is closest to the probability that each of the bins will contain an odd number of balls?

$\textbf{(A)}\ \frac{2}{3} \qquad \textbf{(B)}\ \frac{3}{10} \qquad \textbf{(C)}\ \frac{1}{2} \qquad \textbf{(D)}\ \frac{1}{3} \qquad \textbf{(E)}\ \frac{1}{4}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 20

Not yet answered

Points out of 6

Cyrus the frog jumps 2 units in a direction, then 2 more in another direction. What is the probability that he lands less than 1 unit away from his starting position?

$\textbf{(A)}\ \frac{1}{6} \qquad \textbf{(B)}\ \frac{1}{5} \qquad \textbf{(C)}\ \frac{\sqrt{3}}{8} \qquad \textbf{(D)}\ \frac{\arctan \frac{1}{2}}{\pi} \qquad \textbf{(E)}\ \frac{2 \arcsin \frac{1}{4}}{\pi}$

Select one:

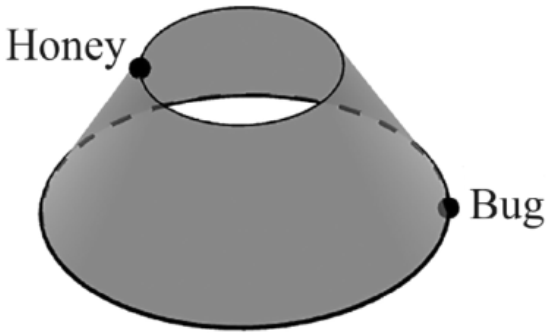
- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 21

Not yet answered

Points out of 6

A lampshade is made in the form of the lateral surface of the frustum of a right circular cone. The height of the frustum is $3\sqrt{3}$ inches, its top diameter is 6 inches, and its bottom diameter is 12 inches. A bug is at the bottom of the lampshade and there is a glob of honey on the top edge of the lampshade at the spot farthest from the bug. The bug wants to crawl to the honey, but it must stay on the surface of the lampshade. What is the length in inches of its shortest path to the honey?



$\textbf{(A)}\ 6+3\pi \qquad \textbf{(B)}\ 6+6\pi \qquad \textbf{(C)}\ 6\sqrt{3}$

$\textbf{(D)}\ 6\sqrt{5} \qquad \textbf{(E)}\ 6\sqrt{3}+\pi$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

A real-valued function f has the property that for all real numbers a and b,

$$f(a+b)+f(a-b)=2f(a)f(b).$$

Which one of the following cannot be the value of $f(1)$?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 1 \qquad \textbf{(C)}\ -1 \qquad \textbf{(D)}\ 2 \qquad \textbf{(E)}\ -2$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 23

Not yet answered

Points out of 6

When n standard six-sided dice are rolled, the product of the numbers rolled can be any of 936 possible values. What is n?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 8 \qquad \textbf{(D)}\ 10 \qquad \textbf{(E)}\ 9$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 24

Not yet answered

Points out of 6

Suppose that a, b, c and d are positive integers satisfying all of the following relations.

$$\begin{aligned} abcd &= 2^6 \cdot 3^9 \cdot 5^7 \\ \operatorname{lcm}(a,b) &= 2^3 \cdot 3^2 \cdot 5^3 \\ \operatorname{lcm}(a,c) &= 2^3 \cdot 3^3 \cdot 5^3 \\ \operatorname{lcm}(a,d) &= 2^3 \cdot 3^3 \cdot 5^3 \\ \operatorname{lcm}(b,c) &= 2^1 \cdot 3^3 \cdot 5^2 \\ \operatorname{lcm}(b,d) &= 2^2 \cdot 3^3 \cdot 5^2 \\ \operatorname{lcm}(c,d) &= 2^2 \cdot 3^3 \cdot 5^2 \end{aligned}$$

What is $\gcd(a,b,c,d)$?

$\textbf{(A)}\ 30 \qquad \textbf{(B)}\ 45 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 6$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**

Question 25

Not yet answered

Points out of 6

A regular pentagon with area $\sqrt{5}+1$ is printed on paper and cut out. The five vertices of the pentagon are folded into the center of the pentagon, creating a smaller pentagon. What is the area of the new pentagon?

$\textbf{(A)}\ 4-\sqrt{5}$ $\textbf{(B)}\ \sqrt{5}-1$ $\textbf{(C)}\ 8-3\sqrt{5}$

$\textbf{(D)}\ \frac{\sqrt{5}+1}{2}$ $\textbf{(E)}\ \frac{2+\sqrt{5}}{3}$

Select one:

- **A**
- **B**
- **C**
- **D**
- **E**
- **Leave blank (1.5 points)**